Course: Mathematical Physics
Problem Set 1
Due: Sept 25, 2019

1. Consider the set $S \subseteq \mathbb{C}$ defined by $\{ e^{\sin \frac{\pi}{t}} \mid t \in (0,1] \}$
   a) Is $S$ open?
   b) Is $S$ closed?
   Give reasons for your answers.

2. The series $\sum_{n=3}^{\infty} \frac{1}{n (\ln n)(\ln(\ln n))^{\alpha}}$, $\alpha \in \mathbb{R}$, is a series of positive terms.
   a) Show that this series either converges or diverges to $\infty$.
   b) Find the condition on $\alpha$ for convergence or divergence of this series.

3. For $u = (u_1, \ldots, u_k) \in \mathbb{R}^k$, the norm (in fact $\ell_2$ norm) $\|u\|$ is defined by $\|u\| = \sqrt{u_1^2 + u_2^2 + \cdots + u_k^2}$.
   Consider $\{ x_n \}$, a sequence of members of $\mathbb{R}^k$. This sequence converges to $x$ if $\forall \varepsilon > 0, \exists N \in \mathbb{N}$, s.t. $n > N \implies \|x_n - x\| < \varepsilon$. This sequence is Cauchy if $\forall \varepsilon > 0, \exists N \in \mathbb{N}$, s.t., $m,n > N \implies \|x_m - x_n\| < \varepsilon$.
   (See sec. 2.4.1 of Vaught).
   Prove that $\{ x_n \}$ is convergent iff it is Cauchy.

Bonus Problem: We discussed Riemann series/rearrangement theorem. You can review the 'algorithm' from the corresponding Wikipedia page.
Write a program to re-arrange $\sum (-1)^k$ to converge to $\pi = 3.14159$... Generate the first 2000 terms or more and plot the partial sums.