1. a) Imagine you observe a source at red-shift $z$ at the present time. Show that in a flat, matter-dominated universe the red-shift changes at a rate:

$$\frac{dz}{dt} = H_0 (1 + z) - H_0 (1 + z)^{3(1+w)/2} \quad (1)$$

b) For what values of $w$ does $z$ decrease with time?

c) For what values of $w$ does $z$ increase with time?

2. The temperature at which the Universe becomes transparent is the temperature when hydrogen becomes stable against photo-dissociation. (This is called “recombination” although it had never “combined” before!) Calculations using the Saha equation put this temperature at 2970 K.

   a) What is the scale factor at this point in time?

   b) What is the redshift?

   c) If the Universe is flat, and matter dominated, what time after the Big Bang does this event occur?

   d) What is the lookback time of this event?

   This event is crucial to our understanding of the history of the Universe, since it represents the earliest direct observational evidence of evolution, and is the source of the surface of last scattering (and all the CMB photons) that are observed with the COBE and WMAP satellites.

3. In class last week, I presented an interesting derivation for a flat, dust-filled Universe of the proper distance of a photon as a function of time that originates on the current particle horizon.

   a) Reproduce that derivation in its entirety.

   b) What is surprising about the photon’s proper distance from us for about 30% of its light travel time?

   c) Comment on how this contrasts with its co-moving coordinate distance.

   (Note: this problem will count for DOUBLE the usual point value, i.e. it will be worth 20 points)