Ph 444 Solutions for Problem Set 2

1. This problem examines a simple model for the distribution of matter in the universe: that the only departures from uniform density are due to the random locations of galaxies (or clusters of galaxies), with the location of each galaxy being uncorrelated with the location of any other. In reality, galaxy positions are correlated: the most likely place to find a galaxy is next to another one.

a) If clumps are placed randomly in space with a uniform average density, then the number of clumps in a volume V will have a Poisson distribution. The probability of seeing i clumps is (see, for example, the CRC Standard Math Tables)

$$p(i) = \frac{e^{-N} N^i}{i!}, \tag{1}$$

where $N = n_c V$ is the average number of clumps in the volume. The only property of the Poisson distribution that is needed for this problem is that the standard deviation about the mean is $\sigma_N = \sqrt{N}$.

b) If each clump has mass m and the clumps are the only mass in the universe, then the average mass in a sphere is $M = mN$ and the standard deviation of that mass around its mean is $\sigma_M = m\sigma_N$. If you want a proof of these statements, let N′ and M′ be the random variables whose means are N and M (in other words, N′ and M′ stand for the observed values for some volume). Also, let $\langle\ \rangle$ stand for the average value. Then $M = \langle M' \rangle = \langle mN' \rangle = m\langle N' \rangle = mN$. Similarly $\sigma_M \equiv \langle (M' - \langle M' \rangle)^2 \rangle^{1/2} = \langle (mN' - \langle mN' \rangle)^2 \rangle^{1/2} = m\langle (N' - \langle N' \rangle)^2 \rangle^{1/2} = m\sigma_N$. With these results,

$$\frac{\sigma_M}{M} = \frac{m\sigma_N}{mN} = \frac{mN^{1/2}}{mN} = \frac{1}{N^{1/2}} = \left(\frac{m}{mN}\right)^{1/2} = \left(\frac{m}{M}\right)^{1/2}. \tag{2}$$

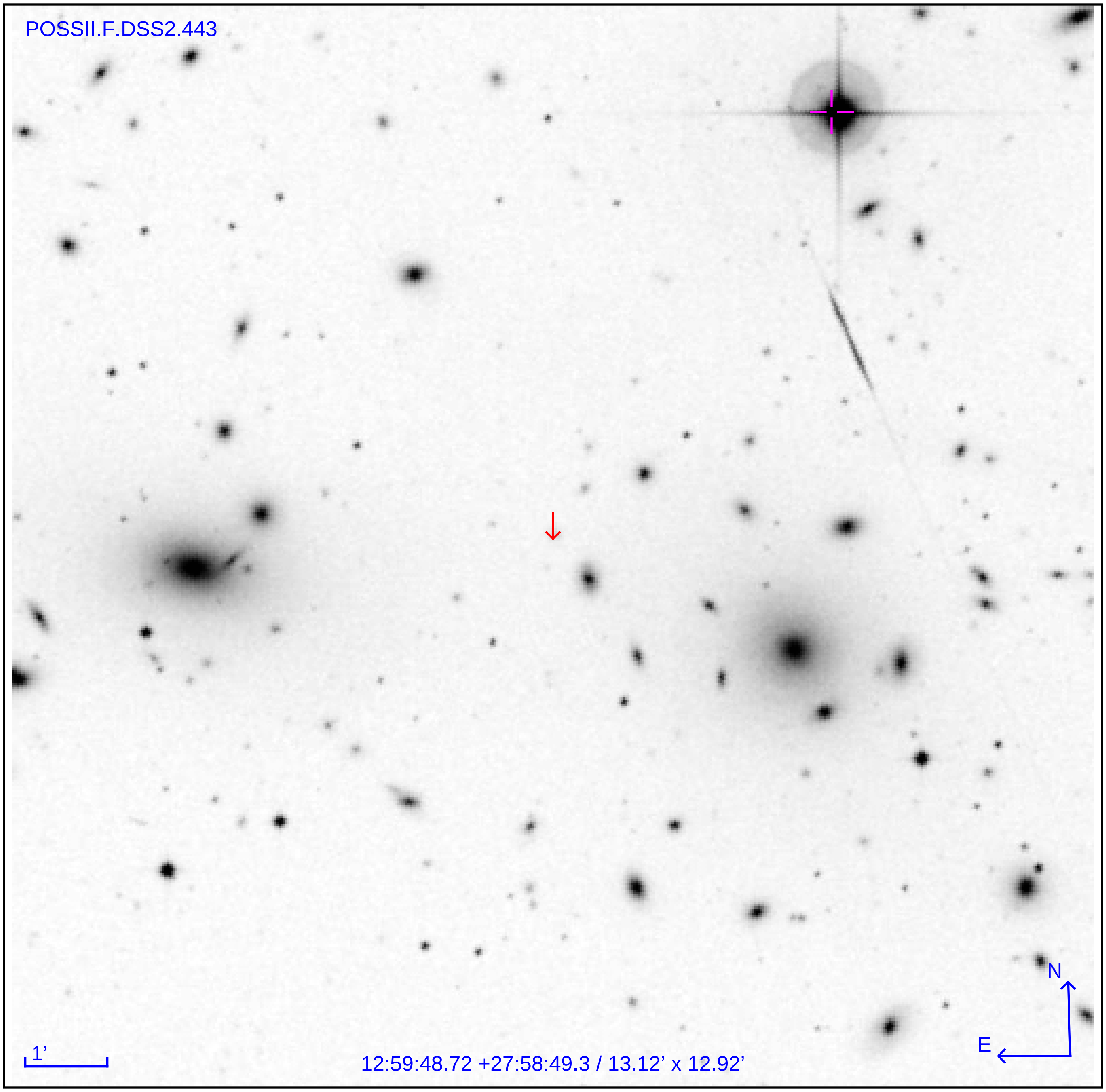
Thus, the random placement of clumps causes the average fractional deviation of the mass in a sphere to decrease more slowly with increasing mass than is observed ($M^{-1/2}$ *versus* $M^{-2/3}$).

c) A sphere with a radius of 100 Mpc has a volume of 4.2×10^6 Mpc^3 and will contain $N = (4.2 \times 10^6\ \mathrm{Mpc}^3)(4 \times 10^{-5}\ \mathrm{clusters/Mpc}^3) = 1.7 \times 10^2$ clusters. The mass in the sphere is then $M = (1.7 \times 10^2)(10^{12}\ M_\odot) = 1.7 \times 10^{14}\ M_\odot$. Then

$$\frac{\sigma_M}{M} = \left(\frac{10^{12}\ M_\odot}{1.7 \times 10^{14}\ M_\odot}\right)^{1/2} = 0.077. \tag{3}$$

A sphere with a radius of 4300 Mpc has $V = 3.3 \times 10^{11}$ Mpc^3, $N = 1.3 \times 10^7$ clusters, $M = 1.3 \times 10^{19}\ M_\odot$, and $\sigma_M/M = 2.7 \times 10^{-4}$. Thus, even in this toy model, the expected deviations from uniform density are small on the scale of the Hubble radius.

2. This lab examined the sky at right ascension $12^h\ 59^m\ 48.7^s$ and declination $+27^\circ\ 58'\ 50''$. This is the center of the Coma cluster of galaxies, the nearest rich cluster to us.



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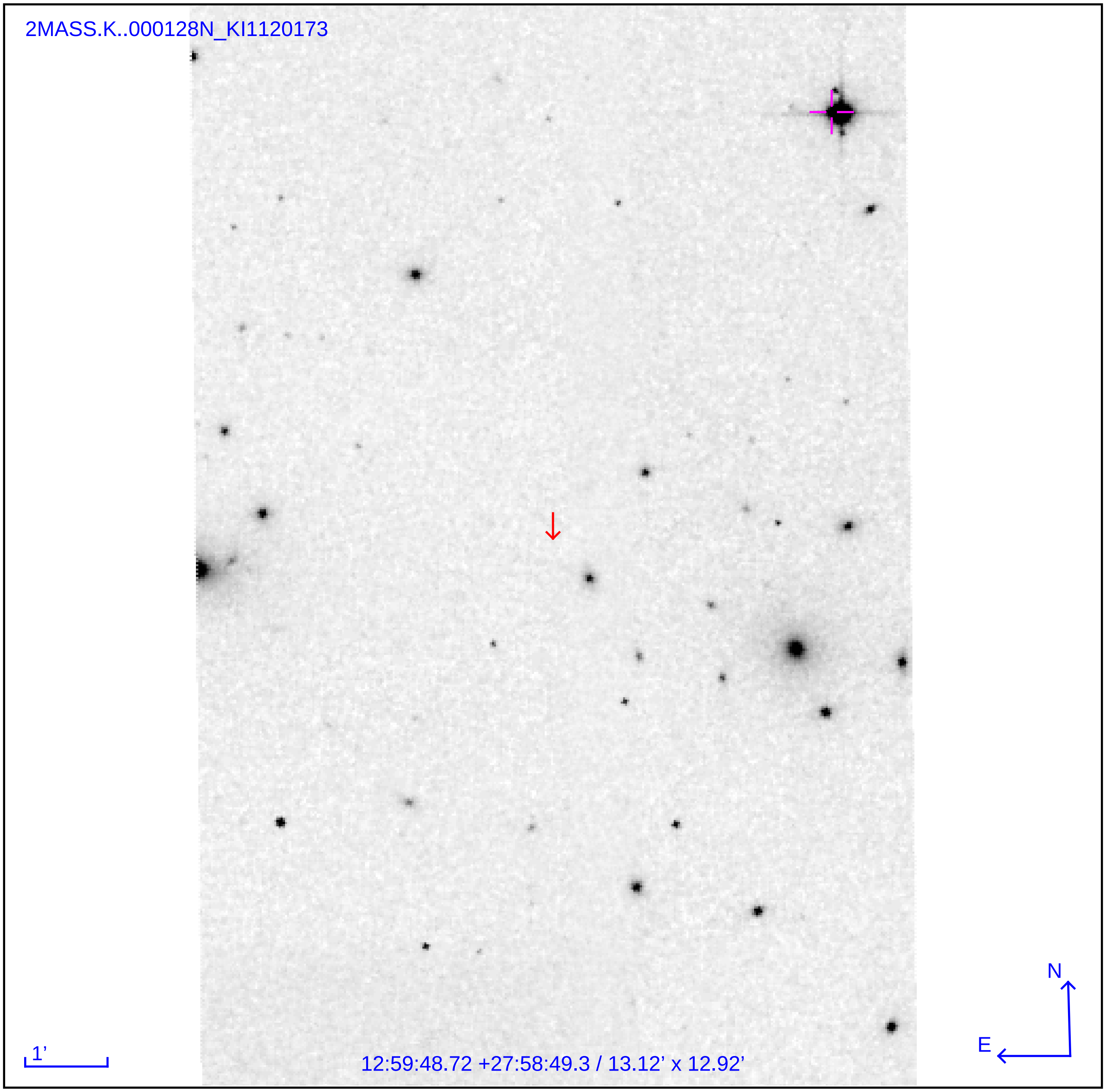

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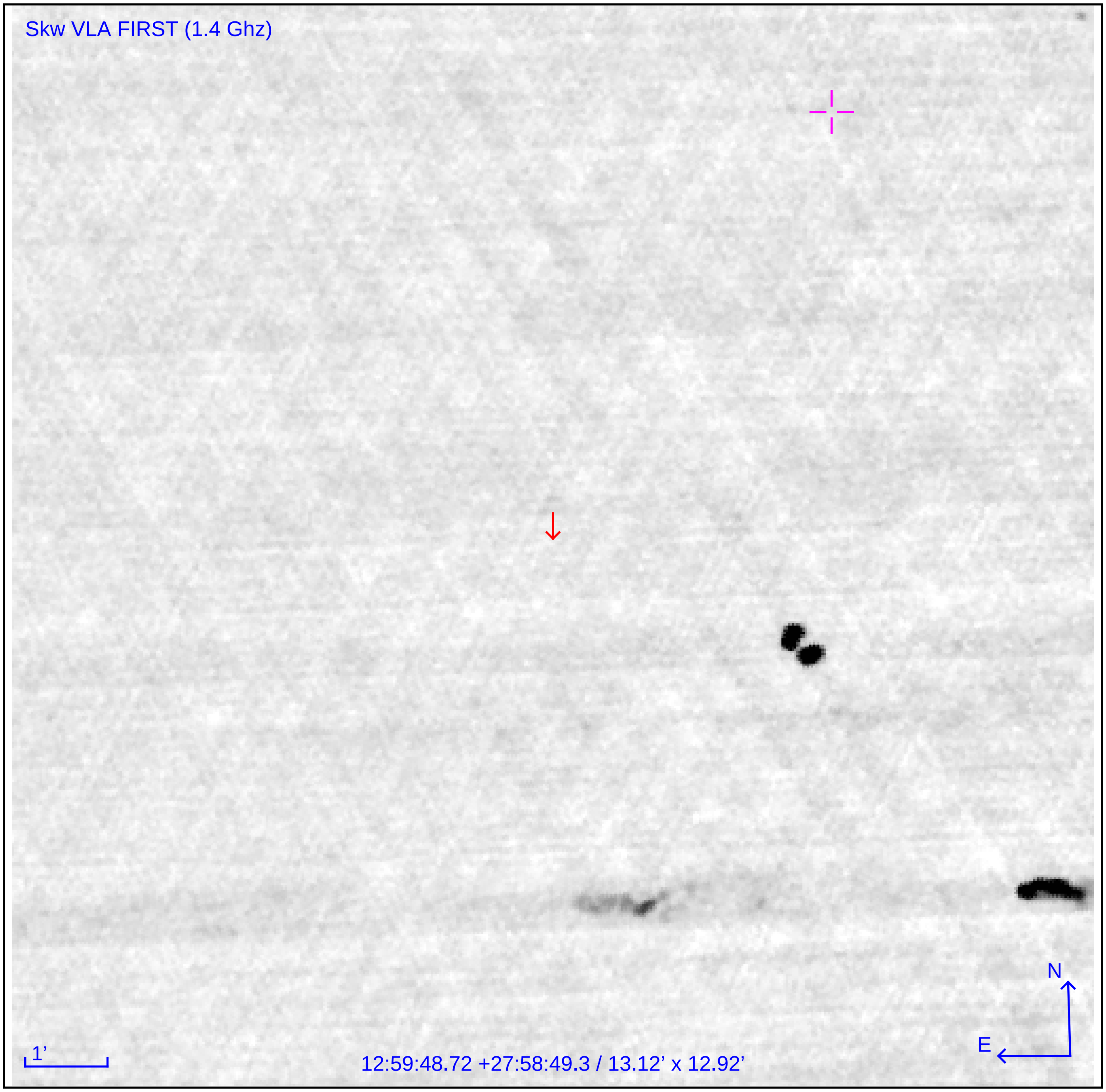

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a) The POSSII image covers 13.1 arcmin $\times$ 12.9 arcmin on the sky, while the 2MASS K image covers 8.6 arcmin $\times$ 17.1 arcmin. They have the same center as nearly as I can tell. Most of the objects in these images are galaxies. The images of the brighter galaxies are "extended", *i.e.*, they are fuzzy blobs instead of points. Some are also not circular in shape. The POSSII image shows more sources, suggesting that it shows fainter objects than the 2MASS image. Blinking between the images using the "blink" image plane created in the next part confirms that the POSSII image shows every object in the 2MASS image, plus fainter objects that do not appear in the 2MASS image. The shallowness of the 2MASS image probably results from the bright emission from Earth's atmosphere at infrared wavelengths and the relatively short exposure times (7.8 s) necessary to image the entire sky with the small infrared imaging detectors.

b) Loading the image from the FIRST (Faint Images of the Radio Sky at Twenty-Centimeters) survey shows only a few bright radio sources in this part of the sky. The two brightest are elliptical blobs of emission that are on either side of the image of the bright galaxy to the right and slightly below the center of the POSSII image. This emission results from two lobes of energetic electrons fed by a jet from the core of the galaxy. There is also some faint, wispy emission near the bottom of the image that may be associated with another galaxy (though it is offset below the galaxy image) or may be an spurious artifact of the numerical techniques used to create this image from the interferometric data. (These data were taken with the Very Large Array interferometer.) The diagonal stripes in the image are definitely artifacts. In the lower right-hand corner of the FIRST image there are 3–4 connected blobs of emission that extend to the right from another galaxy in the POSS image. This emission is from energetic electrons that are swept away from a galaxy that is moving through the core of the cluster by the ram pressure from the hot gas filling the Coma cluster.

c) Marking the entries from the Abell Cluster Catalog on the image and clicking on the single symbol shows that this cluster is number 1656 in the catalog. This catalog resulted from a visual inspection of of the POSSI by George Abell in the 1950's (Abell, 1958, ApJS, 3, 211; some errors in the survey were corrected by Corwin 1974, AJ, 79, 1356). The richness class of this cluster is 2. The richness class is a number between 0 and 5 that indicates the number of galaxies within the cluster (higher numbers mean more galaxies). The catalog also gives a distance class (*i.e.*, an approximate distance) based on the brightness of the 10th brightest galaxy in the cluster.

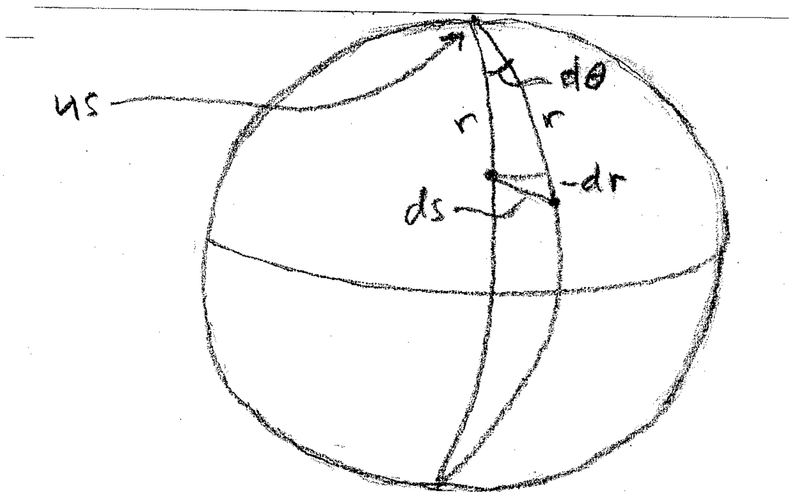
d) The bright galaxy to the right of the image center is PGC 44628. This is the number of this galaxy in the Principal Galaxy Catalog. The line of data displayed from this catalog after clicking on the symbol shows that this galaxy is NGC 4874 (it also has other designations in other catalogs). The bright galaxy to the left of center is PGC 44715 and it is listed as both NGC 4889 and NGC 4884. These two names are because of an error in the original *New General Catalog.* This catalog was compiled by Dreyer in the 1880's based on a heterogeneous set of visual observations.

There were a few hundred errors in the original catalog because of errors in reported locations or plain mistaken observations. Numerous lists of corrections were published in subsequent years, mainly based on photographic images. Dorothy Carlson, one of the numerous unheralded women working in astronomy as assistants during this period, published a large list of corrections in 1940 (Carlson, D. 1940, ApJ, 91, 350). This list notes that NGC 4884 is a duplicate of NGC 4889 based on the inspection of plates taken at the Mount Wilson Observatory and at Heidelberg in Germany.

e) The K magnitude measured by 2MASS for the left-hand bright galaxy is 11.337 ± 0.083. The bright star in the upper-right corner of the POSS image is bright enough to show a halo of reflected light and four bright diffraction spikes. The 2MASS image also shows diffraction spikes. There are two 2MASS point sources at the location of the star. One, which is best centered on the image of the star, has $K=5.984 \pm 0.024$. The other source is a bit to the left and has $K=9.488 \pm 0.086$. The image of the star does seem to bulge out a bit more to the left than to the right. However, I suspect that this is simply an asymmetric response of the 2MASS telescope plus detector to a point source of light rather than an actual second star. Such a bright star may have saturated the detector. Other catalogs based on visible-light images list only a single star at this location. If there is a second star present, it must be very red to only appear at infrared and not optical wavelengths.

3. (Ryden 3.2) This problem examines how the angular diameter of an object of fixed physical size varies with distance in a curved space. For simplicity, we work in two dimensions on the surface of a sphere. Ryden gives the metric for the surface with radius of curvature R:

$$ds^2 = dr^2 + R^2 \sin^2(r/R) d\theta^2. \tag{4}$$



The key to answering this question is to place ourselves at the origin of the coordinate system and then to apply the metric to the separation between the two ends of an object with size ds. As long as $ds \ll R$, this application is accurate. Then the angular diameter which we measure for the object is just $d\theta$ calculated with $dr = 0$. Thus,

$$d\theta = \frac{ds}{R \sin(r/R)}. \tag{5}$$

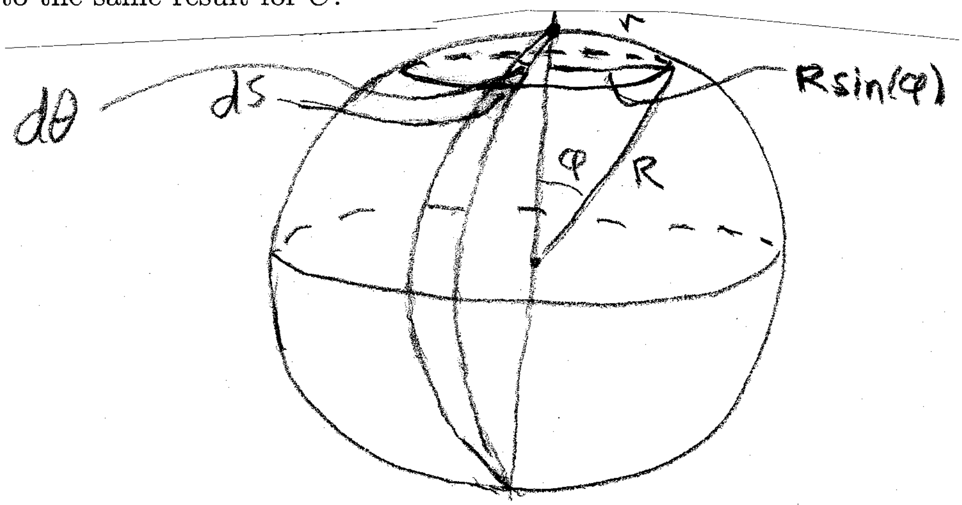
For $r \ll R$, this equation reduces to $d\theta = ds/r$, which is the flat-space result. This makes sense for distances small compared to the radius of curvature. Equation 5 shows that the angular diameter of an object with a fixed size decreases with increasing distance until $r = \pi R/2$, but then increases with increasing distance. This occurs because the distance between any two lines of sight (lines of constant θ) at first increases but then, past the "equator" of the sphere, decreases. As $r \rightarrow \pi R$, $\sin(r/R) \rightarrow 0$, and $d\theta \rightarrow \infty$. The reason for this behavior is that, as an object approaches the opposite pole of the sphere, a fixed ds will cover a wider range of $d\theta$ because of the convergence of the lines of constant θ at the poles. An object right at the opposite pole would be visible along *every* line of sight, making its angular diameter undefined.

4. (Ryden 3.3) This problem examines how easy it is to determine whether a surface is curved using the relation between the radius and circumference of a circle.

There are (at least) two ways to derive the relation between the circumference, C, and radius, r, of a circle on the surface of a sphere. One is to integrate the arclength given by the metric through $\theta = 0$ to 2π while holding r constant. Then

$$\int_0^C ds = \int_0^{2\pi} R\sin(r/R)d\theta \Rightarrow C = 2\pi R\sin(r/R). \qquad (6)$$

Equivalently, one could follow the steps of my derivation of the metric in class. The polar angle of the circle as seen from the center of the sphere is $\phi = r/R$. Then the radius of the circle in the plane which contains it is $R\sin(\phi) = R\sin(r/R)$. This leads to the same result for C.



For $r \ll R$, Equation 6 reduces to the usual flat-space relation between circumference and radius, $C = 2\pi r$. The difference between the two expressions for the circumference is

$$\Delta C = 2\pi R\sin(r/R) - 2\pi r. \qquad (7)$$

One could solve this transcendental equation numerically but, with $\Delta C = 1$ m, it seems reasonable to hope that $r/R \ll 1$ and to expand in that parameter. Then

$$\Delta C = 2\pi R \left(\frac{r}{R} - \frac{1}{3!} \left(\frac{r}{R} \right)^3 + \ldots - \frac{r}{R} \right) \Rightarrow \frac{r}{R} = \left(-\frac{3\Delta C}{\pi R} \right)^{1/3} . \tag{8}$$

The circumference is smaller for a given radius in a positively-curved surface than for a flat surface, though the sign of the difference is unimportant here. The problem left it unclear whether the measurement uncertainty of ± 1 m applied to both the radius and the circumference or just the circumference. However, the difference between the two cases amounts to only a factor of $\sqrt{2}$ in the required ΔC. It is also unclear whether the number is a one-sigma uncertainty or something more significant (say a 95% confidence interval). In a real experiment, a secure confirmation of a curved surface would require at least a 3-sigma difference between the two values for the circumference. I simply adopted $\Delta C = 1$ m $= 1 \times 10^{-3}$ km. With $R = 6371$ km, Equation 8 then yields $r/R = 5.3 \times 10^{-3}$. This justifies using the Taylor expansion. Thus, the circle would need a radius of about 34 km.