Neutron Stars

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5/2/2014
Historical Perspective

- Chadwick discovers neutron in 1932
- Baade & Zwicky link supernovae with stellar collapse to neutron stars 1934
- Discovery of pulsars by Hewish and Bell 1967
Overview

- How are neutron stars similar to white dwarfs?
- Can we come up with a mass limit (like Chandrasekhar mass)
- Observable Properties of neutron stars
Similarities to White Dwarfs

- White dwarfs are supported by electron degeneracy pressure.
- Non relativistic: \( P = K_{NR} \rho^{5/3} \)
  Ultra-Relativistic: \( P = K_{UR} \rho^{4/3} \)
- \( K_{NR} = \frac{h^2}{5m_e} \left( \frac{3}{8\pi} \right)^{2/3} \)
- We can find an upper limit, the Chandrasekhar Mass:
  \[
  M_{CH} = \frac{5.8}{\mu_e^2} M_\odot = 1.4 \ M_\odot \ for \ \mu_e \ of \ 2
  \]
Similarities to White Dwarfs

- At densities below $\sim 10^{17}$ kg/m$^3$ the equation of state is well known
- Use the same logic as before to calculate size/mass of neutron stars:
  - Assume the pressure is coming from ideal gas of relativistic, degenerate neutrons
  - Use the condition of hydrostatic equilibrium
- Use the results from white dwarfs with the change:
  - $\mu_e \rightarrow 1$
Neutron Star Mass and Size

- Equate degeneracy pressure and central pressure required to support the star from HSE:

\[ K_{UR} n_n^{4/3} \approx \left( \frac{\pi}{36} \right)^{1/3} GM^{2/3} \rho_c^{4/3} \]

- For a star of mass \( M \) to be supported by degeneracy pressure of neutrons requires a central density of:

\[ \rho_c \approx 0.91 \left( \frac{M}{M_\odot} \right)^2 \frac{m_n}{(h/m_n c)^3} \]

\[ R \approx 0.77 \left( \frac{M_*}{M} \right)^{1/3} \alpha_G^{-1/2} \frac{h}{m_n c} \]

- Here \( M_* \) and \( \alpha_G \) are related by \( M_* = \alpha_G^{-3/2} m_n \)
Since we did the same analysis with the substitution $\mu_e \rightarrow 1$ we should be able to use our previous result:

$$M_{CH} = \frac{5.8}{\mu_e^2} M_\odot \Rightarrow 5.8 \ M_\odot$$

This is our “zeroth” order estimate

Definitely an upper limit
So how good are our assumptions?

- Our first assumption was to neglect neutron interactions. Not so great assumption at typical neutron star densities
- Another assumption we used implicitly is that gravity is Newtonian. Also not a great assumption
  - Tends to decrease theoretical maximum mass
- Which effect wins?
Including GR/ Assuming an EOS

If we want to include the effects of Einstein’s gravity we need to change our HSE condition:

\[
\frac{dP}{dr} = -\frac{Gm\rho}{r^2} \frac{(1 + P/\rho c^2)(1 + 4\pi r^3 P/mc^2)}{(1 - 2Gm/rc^2)}
\]

And we also need an equation of state

Assuming incompressibility (\(\rho\) constant) we can do a bunch of math to get a mass upper limit of around 5\(M_\odot\)

The unknown here is the equation of state

Calculations give a range of 1 – 3 \(M_\odot\) for the upper limit of neutron stars
Some Observable Stuff

- Hewish & Bell 1967 discover a 1.33 second radio pulse
- 1968 Crab Pulsar is discovered $\tau = 0.33$ ms
- Could this be a star (white dwarf?) producing this signal?

$$\tau_{\text{min}} = 2\pi \left( \frac{R^3}{GM} \right)^{1/2}$$

- Typical numbers for a white dwarf give a minimum period of $\sim 10$ s
Where does the pulse come from?

- Rapid rotation of the neutron star + magnetic field → magnetic dipole radiation
- The rotation of the Crab Pulsar is slowing down (it is radiating) at a rate:
  \[
  \frac{d\omega}{dt} = -2.4 \times 10^{-9} \text{s}^{-2}
  \]
- Corresponding energy loss rate:
  \[
  \frac{dE}{dt} = I\omega \frac{d\omega}{dt}
  \]
- Putting in numbers gives the energy loss rate of the Crab Pulsar to be \(4.6 \times 10^{31}\) W
- The luminosity of the Crab Nebula is around \(5 \times 10^{31}\) W.
Estimating the B field

If we assume the Crab Nebula luminosity comes from the pulsar’s magnetic dipole radiation

\[ P_{B_{\text{dipole}}} = \frac{2}{3c^3} \frac{\mu_0}{4\pi} m^2 \omega^4 \sin^2 \theta \]

Then we can estimate the magnetic field at the surface of the neutron star:

\[ B \approx \frac{\mu_0 m}{4\pi R^3} \sim 10^8 \text{ T} \]