**Problem 1 (4 points).**

(a) (1 point) Where is the chemical potential $\mu$ located in an intrinsic semiconductor at $T = 0$?

(b) (2 point) Does $\mu$ vary with temperature in an intrinsic semiconductor? Provide an explanation based on the densities of states in the conduction and valence bands.

(c) (1 point) How does the position of $\mu$ change if the semiconductor is doped with (i) donors (ii) acceptors?

(d) (1 point) Where is $\mu$ located in a *degenerate* acceptor-doped semiconductor?
Problem 2 (5 points). Silicon is doped with donors, \( N_D = 10^{23} \text{m}^{-3} \). This donor concentration is much smaller than \( N_D = 10^{25} \text{m}^{-3} \) that corresponds to the degenerate case. Assume that all donors are ionized at room temperature. It is also given that \( n_i^2 = 10^{31} \text{m}^{-6} \).

What are the electron and hole concentrations at (a) 300K, and (b) 0K? Provide short explanations.

(a) 300K

\[
n = \frac{1}{2} \left[ N_D + \sqrt{(N_D)^2 + 4n_i^2} \right] \approx 10^{23} \text{m}^{-3}
\]

\[
p = \frac{1}{2} \left[ -N_D + \sqrt{(N_D)^2 + 4n_i^2} \right] \approx \frac{n_i^2}{N_D} = 10^8 \text{m}^{-3}
\]

(b) 0K  \( n = p = 0 \) “freeze-out” of carriers
Problem 3 (2 points). The direct band gap in Germanium is 0.8 eV. Explain why Ge is opaque to visible light. What is the longest light wavelength to which Ge becomes transparent?

\[ h\nu = \frac{hc}{\lambda} = E_g \]

\[ \lambda = \frac{hc}{E_g} = \frac{6.6 \times 10^{-34} \cdot 3 \times 10^8}{0.8 \cdot 1.6 \times 10^{-19}} = 1.55 \mu m \]
Problem 4. (4 points) Consider a cylinder made of a type-II superconductor; the cylinder is in an external field $B$ pointing along the cylinder axis.

(a) (2 points) Sketch all currents in the cross section of the cylinder at $B < B_{C_1}$ and $B_{C_1} < B < B_{C_2}$. Indicate current and field directions.

(b) (2 points) Calculate the areal density of vortices for $B^* = 1T$ ($B_{C_1} < B^* < B_{C_2}$).

\[
\text{density of vortices} = \frac{B \times 1 m^2}{\Phi_0} = \frac{1}{2 \times 10^{-15}} = 5 \times 10^{14} m^{-2}
\]
Problem 5. (6 points) Consider an $n$-type Silicon MOSFET with the width of the conduction channel $W = 10\mu m$, its length $L = 1\mu m$, the gate oxide thickness $t_{ox} = 20nm$ ($\epsilon_{SiO2} = 4$), the electron mobility $0.03m^2/Vs$, and the threshold voltage $V_T =$ 1V. $\varepsilon_0 \approx 9 \times 10^{-12} C/Vm$.

(a) (1 point) Calculate the gate-channel capacitance per unit area.
(b) (2 points) Calculate the drain-source current for the gate voltage $V_G = 3V$ and the drain-source voltage $V_{SD} = 0.1V$.
(c) (1 point) If the gate voltage is fixed at $3V$, at which $V_{SD}$ the MOSFET enters the saturation regime?
(d) (2 points) Calculate the drain-source current in the saturation regime for the gate voltage $V_G = 3V$.

\[
(a) \quad C_{ox} = \frac{\varepsilon_0 \varepsilon}{t_{ox}} = \frac{4 \times 9 \times 10^{-12}}{20 \times 10^{-9}} = 1.8 \times 10^{-3} F/m^2
\]

\[
(b) \quad I_{DS} = \mu C_{ox} (V_{GS} - V_T) \frac{W}{L} V_{DS} = 1.08 \times 10^{-4} A
\]

\[
(c) \quad V_{DS} > 2V
\]

\[
(d) \quad I_{DS,sat} = \mu C_{ox} \frac{W (V_{GS} - V_T)^2}{2L} = 1.08 \times 10^{-3} A
\]
Problem 6. (5 points) The temperature dependence of the charge-carrier concentration for a certain n-doped semiconductor is shown in the Figure.
(a) (3 points) Estimate the bandgap for the semiconductor and the concentration of donor ions.
(b) (2 points) Describe an experimental method by which this data could have been measured.