Modern Physics: Nuclear Power (Part I)

Physics of Modern Devices
March 23, 2020
Outline

- **Today: Part I**
  - Overview Nuclear Physics/Nuclear Structure
  - Some other devices which use nuclear physics

- **On Wednesday: Part II**
  - Nuclear Fission Weapons
  - Nuclear Fusion Weapons
  - Radiation levels
  - Radiation detectors
Atomic Nucleus

- Atoms are electrically neutral:
  - They have as many protons (+) as electrons (−), so charge is compensated.

- At the center of an atom is its nucleus (Rutherford, α particle scattering)
  - Extremely small (1/100,000th of atom’s diameter);
  - Contains most of the atom’s mass;
  - Also contains most of the atom’s potential energy;
    - Evidence is related to: $E=mc^2$
The mass of an element that is numerically equal to the atomic mass $A$ in grams is called a mole and will contain Avogadro's number $N_A$ of nuclei.

Example: carbon

$$^{12}_6 \text{C}$$

$A = 12$ nucleons ($p$ and $n$),
$Z = 6$ protons,
$12 - 6 = 6$ neutrons.
If you take an Avogadro's number \( N_A = 6.022 \times 10^{23} \text{ mole}^{-1} \) of atoms or any element (that is, 1 mole of it), the mass of such a sample will be \( A \) grams.
## Masses

<table>
<thead>
<tr>
<th>Particle</th>
<th>kg</th>
<th>u</th>
<th>MeV/c^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proton</td>
<td>1.673x10^{-27}</td>
<td>1.00728</td>
<td>938.28</td>
</tr>
<tr>
<td>Neutron</td>
<td>1.675x10^{-27}</td>
<td>1.00867</td>
<td>939.57</td>
</tr>
<tr>
<td>Electron</td>
<td>9.11x10^{-31}</td>
<td>5.49x10^{-4}</td>
<td>0.511</td>
</tr>
</tbody>
</table>

1 u = M(^{12}\text{C})/12 = 1.66x10^{-27} kg = 931.5 MeV/c^2

Protons or neutrons are about 2000 times heavier than electrons.
Structure of Nucleus

Nucleus contains two kinds of nucleons:
- Protons are positively charged
- Neutrons are neutral

Two forces are active in a nucleus:
- Electrostatic repulsion between protons
- Nuclear force attraction between touching nucleons
- Short distances: nuclear force stronger than electric
- Long distances: electric force stronger than nuclear
Scales: Nuclear Size

- Relative scale model of an atom and the solar system

- Relative size of nuclei

So, atoms are mostly an empty space!
Nuclear Size and Density

- Scattering experiments suggest that nuclei are roughly spherical and appear to have essentially the same density.

\[ r = r_0 A^{1/3} \] 
where \( r_0 = 1.2 \times 10^{-15} \text{ m} = 1.2 \text{ fm} \)

- \( r \) is the radius of the nucleus of mass number \( A \)
- nuclear density seems to be independent of the details of neutron number or proton number
  - implies that the force between the particles is essentially the same whether they are protons or neutrons.
  - the strong force is the same between any pair of nucleons.

One fermi (f) = 10^{-15} m
The strong force between two protons is about the same as the strong force between two neutrons, or a proton and a neutron.

Beyond about one fermi the strong force declines extremely rapidly.

As more protons are added to the nucleus, more neutrons are needed to bind the protons together, but the larger the nucleus becomes, the farther apart are the protons and the less effective is the strong force.
Electric force is longer range than the strong force.

Eventually separation becomes too great for the strong force to compensate for the repulsive forces.

Nuclei spontaneously disintegrate for proton numbers larger than 83 (radioactivity).

Unstable nuclei!

The release of light and or particles which accompanies the disintegration is called radiation or radioactivity.
The larger the binding energy of a nucleus, the more stable it is. The binding energy is the difference between the rest energies. Nuclear binding energy = $\Delta mc^2$

$\Delta m = 0.0304 \text{ u}$ which gives a binding energy of 28.3 MeV for Helium (alpha particle)

There are four nucleons, so the binding energy per nucleon is about 28/4, or about 7 MeV per nucleon.
Binding Energy

- The binding energy curve is obtained by dividing the total nuclear binding energy by the number of nucleons.

- The fact that there is a peak in the binding energy curve in the region of stability near iron means that either the breakup of heavier nuclei (fission) or the combining of lighter nuclei (fusion) will yield nuclei which are more tightly bound (less mass per nucleon).

- Nuclei with the largest binding energy per nucleon are the most stable.

- The largest binding energy per nucleon is 8.7 MeV, for mass number $A = 60$.

- Iron group most tightly bound

- Beyond bismuth, $A = 209$, nuclei are unstable.
Problem

- What is the binding energy of a deuteron?
- A deuteron consists of a proton and a neutron and is the nucleus of deuterium (also called heavy hydrogen).
- Mass of the deuteron is 2.0136 u

\[
\begin{align*}
m_d &= 2.0136u \\
m_p &= 1.00728u \\
m_n &= 1.00867u \\
m_p + m_n &= 2.01595 \\
\Delta m &= (m_p + m_n) - m_d = 2.35 \times 10^{-3} u = 2.2 MeV
\end{align*}
\]
Nuclear Stability

- In a nucleus, nucleons are in equilibrium
- To be classically stable, equilibrium must be stable
- To be quantum-mechanically stable, equilibrium must also be the potential energy minimum
- Quantum mechanics and the Heisenberg uncertainty principle allow the nucleons to try out arrangements outside their equilibrium positions
- If they find a path to a new equilibrium, they may take it and the nucleus may fall apart
Uncertainty Principle

- The position and momentum of a particle cannot be simultaneously measured with arbitrarily high precision.
  - There is a minimum for the product of the uncertainties of these two measurements.
  - There is likewise a minimum for the product of the uncertainties of the energy and time.

\[
\Delta x \Delta p > \frac{\hbar}{2} \\
\Delta E \Delta t > \frac{\hbar}{2}
\]

\[
\hbar = h/2\pi = 1.055 \times 10^{-34} \text{ J} \cdot \text{s}
\]

- It arises from the wave properties inherent in the quantum mechanical description of particles.
  - Wave-Particle Duality

- *Even with perfect instruments and technique, the uncertainty is inherent in the nature of things.*
Confinement

The uncertainty principle contains implications about the energy which would be required to contain a particle within a given volume.

Assume atomic size = 4 Å

Nuclear size = \( \frac{1}{20,000} \times 4 \, \text{Å} \)

Using the atomic size as the uncertainty in position

\[ \Delta p = \frac{\hbar}{\Delta x} = 1.66 \times 10^{-24} \, \text{kg m/s} \]

Assume \( \Delta p = p \)

and \( E = \frac{p^2}{2m} \)

Energy to:

- Confine electron in atom: 9.4 eV
- Confine proton in nucleus: 2.05 MeV
- Confine electron in nucleus: 3.77 GeV

These are in the range of observed atomic and nuclear processes.

This is about a factor of a thousand above the observed energies of nuclear processes, indicating that the electron cannot be confined in the nucleus.
Confinement

Confinement in atom
Assume atomic size = 4 Å = Δx

\[ Δp = \frac{h}{Δx} = 1.66 \times 10^{-24} \text{ kg m/s} \]

\[ Δp = p, \quad E = \frac{p^2}{2m} \]

For electron:

\[ E = \frac{(1.66 \times 10^{-24} \text{ kg m/s})^2}{2(9.11 \times 10^{-31} \text{ kg})(1.6 \times 10^{-19} \text{ J/eV})} \]

\[ E = 9.4 \text{ eV} \]

Confinement in nucleus
Nuclear size = \( \frac{1}{20,000} \times 4 \text{ Å} = Δx \)

\[ Δp = \frac{h}{Δx} = 3.31 \times 10^{-20} \text{ kg m/s} \]

For electron:

\[ E = \frac{(3.31 \times 10^{-20} \text{ kg m/s})^2}{2(9.11 \times 10^{-31} \text{ kg})(1.6 \times 10^{-19} \text{ J/eV})} \]

\[ E = 3.77 \times 10^9 \text{ eV} = 3.77 \text{ GeV} \]

For proton, divide by \( m_p/m_e = 1836 \):

\[ E = 2.05 \times 10^6 \text{ eV} = 2.05 \text{ MeV} \]
Radioactivity

- Protons repel & neutrons are unstable
- Large nuclei have two problems:
  - Too many protons, too much electrostatic potential
  - Too many neutrons, then neutrons are unstable
  - Delicate balance between protons and neutrons
- Large nuclei tend to fall apart spontaneously
- Such decay is called fission
- Fission is a type of radioactivity
  - More about this next time
Radioactive Decay

- The most common types of radiation are called alpha, beta, and gamma radiation.
- Radioactive decay rates are normally stated in terms of their half-lives.
Half-Life

- The radioactive half-life for a given radio-isotope is the time for half the radioactive nuclei in any sample to undergo radioactive decay.
- After two half-lives, there will be one fourth the original sample, after three half-lives one eight the original sample, and so forth.

\[ N = N_0 e^{-\lambda t} \]

- \( N \): Number of radioactive nuclei
- \( N_0 \): Number of radioactive nuclei at \( t = 0 \)
- \( \lambda \): decay constant
- \( t \): time

\[ R = \left| \frac{dN}{dt} \right| = N_0 \lambda e^{-\lambda t} = R_0 e^{-\lambda t} \]

- \( R \): decay rate

\[ T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda} \]

- \( T_{1/2} \): half-life
Radioactive Dating

- Carbon-dating:
  - dating of organic (once living) materials
  - matter takes in carbon dioxide from the air for photosynthesis.

- Process:
  - Cosmic ray protons blast nuclei in the upper atmosphere, producing neutrons which in turn bombard nitrogen.
  - This neutron bombardment produces the radioactive isotope carbon-14.
  - The radioactive carbon-14 combines with oxygen to form carbon dioxide and is incorporated into the cycle of living things.
  - The carbon-14 forms at a constant rate, so that by measuring the radioactive emissions from once-living matter and comparing its activity with the equilibrium level of living things, a measurement of the time elapsed can be made.
How many nuclei are left?

Carbon 14 is a radioactive isotope of carbon and its half-life is 5730 years. If you start with a sample of 1000 carbon-14 nuclei, how many will still be around in 22,920 years?

In 5730 years (one half-life), half the sample decayed leaving 500 $^{14}_6C$ nuclei remaining.

\[
N = 1000e^{-\frac{0.693}{T_{1/2}} t} = 1000e^{-\frac{0.693}{5730} \times 22920} = 1000e^{-\frac{0.693}{5730} \times 5730} = 500, \text{ for } t = T_{1/2}
\]

\[
\frac{22920}{5730} = 4 \text{ half-lives}
\]

\[
N = 1000e^{-0.693 \times 4} = 1000 \cdot 0.0625 = 62.5
\]

So, \(N \approx 62\text{nuclei}\)
Problem

Radioactive dating

A piece of charcoal of mass 25g is found in some ruins in an ancient city. The sample shows a carbon-14 activity of 250 decays/min. How long has the tree that this charcoal came from been dead?

Assume ratio of carbon-14 to carbon-12 in atmosphere is $1.3 \times 10^{-12}$
\[ ^{14}\text{C}: \lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{5730\text{yrs} \cdot 3.15 \times 10^7 \text{s/yr}} = 3.84 \times 10^{-12} \text{s}^{-1} \]

Number of \(^{12}\text{C}\) nuclei in 25g of carbon:

\[ N^{^{12}\text{C}} = \frac{6.022 \times 10^{23} \text{nuclei/mol}}{12 \text{g/mol}} \times 25 \text{g} = 1.25 \times 10^{24} \text{nuclei} \]

\[ N_0^{^{14}\text{C}} = 1.3 \times 10^{-12} \times 1.25 \times 10^{24} = 1.63 \times 10^{12} \text{nuclei} \]

\[ R_0 = N_0 \lambda = 1.63 \times 10^{12} \times 3.84 \times 10^{-12} \text{s}^{-1} = 6.26 \text{decays/ sec} = 376 \text{decays/ min} \]

\[ R = R_0 e^{-\lambda t} \Rightarrow \frac{R}{R_0} = e^{-\lambda t} \Rightarrow -\lambda t = \ln\left(\frac{R}{R_0}\right) \]

\[ -\lambda t = \ln\left(\frac{250}{376}\right) = -0.408 \]

\[ t = \frac{0.408}{3.84 \times 10^{-12} \text{s}^{-1}} = 1.06 \times 10^{11} \text{s} = 3370 \text{years} \]
Real Examples

- The Iceman:
  - Found in Italian Alps in 1991
  - remains are about 5700 years old.

- The Shroud of Turin:
  - In 1988, three laboratories all agreed that the shroud was 608-728 years old
**Alpha Decay**

- Nucleus emits an alpha particle
- Loses two protons and two neutrons

\[
A^Z_X \rightarrow A^{\text{He}}_2 + Y^{-4}_{Z-2}
\]
Smoke Detectors

- The source of ionizing radiation is a minute quantity of americium-241 (~ 1/5000th of a gram), which is a source of alpha particles.
- The ionization chamber consists of two plates separated by about a centimeter.
- The battery applies a voltage to the plates, charging one plate positive and the other plate negative.
- Alpha particles constantly released by the americium knock electrons off of the atoms in the air, ionizing the oxygen and nitrogen atoms in the chamber.
- The positively-charged oxygen and nitrogen atoms are attracted to the negative plate and the electrons are attracted to the positive plate, generating a small, continuous electric current.
- When smoke enters the ionization chamber, the smoke particles attach to the ions and neutralize them, so they do not reach the plate.
- The drop in current between the plates triggers the alarm.
Beta Decay

- Beta particles are just electrons from the nucleus.
- The daughter nucleus has the same number of nucleons as the parent nucleus but charge number changes by 1.

\[
\begin{align*}
\frac{A}{Z} X & \rightarrow \frac{A}{Z+1} Y + \beta^- \\
\frac{A}{Z} X & \rightarrow \frac{A}{Z-1} Y + \beta^+
\end{align*}
\]

Two typical beta decay processes:

- \( ^{14}_6 \text{C} \rightarrow ^{14}_7 \text{N} + \beta^- \)
- \( ^{12}_7 \text{N} \rightarrow ^{12}_6 \text{C} + \beta^+ \)
Gamma Decay

- Gamma radioactivity is composed of electromagnetic rays.
- Nucleus in excited state
  - Violent collision with another particle
  - After it has undergone alpha or beta decay

\[ ^A_Z X^* \rightarrow ^A_Z X + \gamma \]

- More applications of this type of radioactivity next time!
Radiation

- Alpha:
  - Range is less than a tenth of a millimeter inside the body
  - Its main radiation hazard comes when it is ingested into the body
  - It has great destructive power within its short range
    - In contact with fast-growing membranes and living cells, it is positioned for maximum damage.
  - Not suitable for radiation therapy

- Beta:
  - High energy electrons have greater range of penetration than alpha particles, but much less than gamma rays
  - The radiation hazard from betas is greatest if they are ingested

- Gamma:
  - Most gamma rays are high energy and very penetrating
  - It is the most useful type of radiation for medical purposes
  - But the most dangerous
    - Ability to penetrate large thicknesses of material
Announcements

- Next time:
  - Fusion/Fission and more …

- Homework 7:
  - Due Wed. 3/25 via upload of a pdf file online (see *Homeworks* page on our regular course’s web site)

- Use our regular course’s web site at Physics for updates, assignments, announcements.

- Presentations:
  - I plan to post the tentative schedule by Friday.