Problem 1

\[
\frac{1}{2\pi \sqrt{LC}} \Rightarrow f^2 = \frac{1}{(2\pi f)^2} \cdot \frac{1}{LC}
\]

Resonance freq. (in Hz)

of LC circuit

\[
\frac{dC}{df} = \frac{1}{L} \cdot (-2) \cdot \frac{1}{(2\pi f)^3} \cdot 2\pi
\]

\[
\Delta C = \frac{dC}{df} \cdot \Delta f
\]

\[
\frac{\Delta C}{C} = \left( \frac{dC}{df} \right) \cdot \frac{\Delta f}{C} = -\frac{2}{L} \cdot \frac{1}{(2\pi f)^2} \cdot \frac{\Delta f}{f} \cdot \frac{1}{C} = -2 \cdot \frac{\Delta f}{f}
\]

\[
\Delta f = 0.1 \text{ MHz}
\]

\[
f \approx 100 \text{ MHz}
\]

\[
\frac{\Delta C}{C} = -2 \cdot \frac{0.1}{100} = -0.002 = 0.2\%
\]

Hence to resolve \(\Delta f = 0.1 \text{ MHz}\) at \(f = 100 \text{ MHz}\), one needs capacitance tuning at accuracy of \(\frac{\Delta C}{C} < 0.2\%\) or better. Radio with 1\% accuracy of tuning \(C\) will not allow to resolve these.
Problem #2

Radio #1 frequency range:

\[ f_1 = \frac{1}{2\pi} \sqrt{\frac{1}{0.5 \times 10^{-3} \times (20 \div 2000) \times 10^{-12}}} = \]

\[ = 158.2 \text{ kHz} \div 1.59 \text{ MHz.} \quad (\text{range 1}) \]

Radio #2 frequency range:

\[ f_2 = \left(\frac{1}{2\pi}\right)^{1/2} \sqrt{\left(10^{-6} \div 10^{-4}\right) \times 36 \times 10^{-12}} = \]

\[ = 2.65 \text{ MHz} \div 26.5 \text{ MHz.} \quad (\text{range 2}) \]

(a) **200 kHz** is in range 1, so **Yes**.

(b) **2 MHz** is in the gap between ranges 1 & 2, so **No**.

(c) **20 MHz** is in the 2nd range, so **Yes**.
Problem #3

Besides the two ranges, 159.2 kHz - 1.59 MHz and 2.65 MHz - 26.5 MHz, available with the original two radios, we can create a combination of \( L_1 \) and \( C_2 \) (both fixed) that would give us a single frequency:

\[
f_{(L_1 C_2)} = \frac{1}{2\pi} \frac{1}{\sqrt{L_1 C_2}} = \frac{1}{6.28} \frac{1}{\sqrt{0.5 \times 10^{-3} \times 36 \times 10^{-12}}} = \]

\[
= 1.18 \text{ MHz} \quad (\text{not helpful for (a,b,c)})
\]

We can also combine \( L_2 \) and \( C_1 \) (both variable) that can give us a wider range of accessible frequencies:

\[
f_{(L_2 C_1)} = \frac{1}{2\pi} \frac{1}{\sqrt{L_2 C_1}} = \frac{1}{6.28} \frac{1}{\sqrt{(10^{-6} + 10^{-4}) \times (20 + 2000) \times 10^{-12}}} =
\]

\[
= 35.6 \text{ kHz} \div 35.6 \text{ MHz}.
\]

So, for (a) we cannot listen to 180 kHz,
for (b) we can listen to 2 MHz,
for (c) we cannot listen to 40 MHz.