The Proper Lifetime of a Muon

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The lifetime of the muon is measured twice and compared with the accepted value. The ratio of the number of positively charged muons to negatively charged muons is calculated. From the measured lifetime of the muon, the Fermi coupling constant is calculated and compared with the accepted value. The number of false muon decays is estimated and compared against the total number of measured decays.

Introduction

The muon was first detected in 1936 by Carl Anderson in his study of cosmic radiation. This was done at a time when experiment was far enough ahead of theory that theoretician I. I. Rabi reportedly said, in response to the discovery of the muon, "Who ordered that?" The muon is a fundamental particle with mass approximately 200 times that of an electron, the same charge as an electron, and lifetime of about $2.197\ \mu s$.[1] This lifetime is small enough that the number of muons that reach the ground from cosmic rays (see below) would be completely negligible if classical mechanics were correct. However, special relativity predicts that many muons should make it to earth due to time dilation. Time dilation was first measured experimentally in 1941 on Mount Washington using muons.

Background

The muons used in this experiment were produced from cosmic rays. Incoming cosmic rays (usually protons) collide with atoms in the upper atmosphere creating pions which then decay into (mainly) muons and neutrinos. It is these secondary muons that will be the source.

The primary decay mode for the muon is:

$$\mu^- \rightarrow e^- + \bar{\upsilon}_e + \upsilon_\mu$$

i.e., a muon decays into an electron, an antielectron-neutrino, and a muon-neutrino.

It is observed that muons penetrate much further into matter than electrons. This is because of the great mass difference between the two particles; muons emit much less bremsstrahlung radiation due to the decreased ability of atomic electromagnetic fields to slow the muons to capture velocities. Once they are slowed, they decay as described above. It is important to note that they are slowed to non-relativistic speeds, and because of this, the muon lifetime measured in this experiment will not need to be corrected for time dilation effects.

The lifetime of the muon is, however, impacted by its interaction with matter. The positive muon decays as usual, but the negative muon has another mode of decay in matter. The negative muon has an atomic bound state, which is not excluded by the Pauli principle as the muon is not an electron. These bound muons can interact with a proton in the nucleus via:

$$\mu^- + p \rightarrow n + \upsilon_\mu$$

i.e. a negative muon interacts with a proton to produce a neutron and a muon-neutrino. Because of the existence of this other decay process, the negative muon has a shorter lifetime in matter. In this experiment, because the apparatus cannot distinguish the types of decays, the observed muon lifetime will be the weighted average of τ^+ and τ^- where τ^+ and τ^- are the lifetimes of the positive and negative muons respectively.

There is another measure of the muon lifetime, since the lifetime is related to the Fermi coupling constant G_F by[2]:

$$\tau = \frac{192\pi^3\hbar^7}{G_F^2 m^5 c^4}$$

Where τ is the muon lifetime, ħ is Planck's constant divided by 2π, m is the muon mass, and c is the speed of light in vacuum. This formula will later be used to calculate G_F.

Detection

The lifetime of the muon was measured in two similar, but slightly different, ways. First, a commercially available muon detection apparatus, built by TeachSpin, was used and various experiments were performed with it. Second, a homemade muon detector was built using four photomultiplier tubes (PMT), a homemade logic circuit, and a TAC (time to amplitude converter). The TeachSpin apparatus will be discussed in detail below.

The homemade apparatus consisted of four PMT's that, when wired together in different ways provided either a start or stop signal to the TAC. Extra cable length was added to two of the PMT's in order to correct for slight delays (on the order of 40 ns) that were added by the logic circuits. The TAC converted different delay times between the start and stop signals into a voltage pulse of a proportional amplitude. The signal from the TAC was then fed through a multi-channel analyzer (MCA) on the computer that created a histogram of the number of counts per energy channel, which was later calibrated by feeding the TAC two signals separated by a known delay time (as measured by an oscilloscope).

The TeachSpin apparatus and the experiments conducted with it will be discussed first, followed by the homemade apparatus.

One last point must be made about false decays. Because it is possible that the start and stop signals registered by the apparatus may actually be caused by two different muons that happen to be incident at nearly the same time, there will be background noise in the measurements. This background will be discussed in more depth below, and in particular the rate of false muon decays will be compared to the rate of measured muon decays.

Characterization of the TeachSpin Apparatus

Before the TeachSpin apparatus was used to measure muon decay time, various tests were run in order to characterize its internal electronics and to become more familiar with the apparatus.

The gain and frequency response of the internal amplifier of the apparatus was measured first. A function generator was used to input a 100 kHz, 100 ± 2 mV peak-to-peak sine wave to the device and the output of the amplifier was read on an oscilloscope. The resulting output of the amplifier was observed to be a sine wave of the same frequency with a peak-to-peak amplitude of 1.75 ± 0.05 V, corresponding to a gain of 35 ± 2.

The frequency response of the amplifier was then observed by using the same input wave to the apparatus and continuously increasing its frequency while observing the resultant output of the amplifier on an oscilloscope. The gain was observed to remain constant until the input wave reached a frequency of 200 kHz, at which point it began to decrease. The gain continued to decrease in a continuous fashion as the frequency of the input wave was increased.

The saturation output voltage of the amplifier was then measured by inputting a 100 kHz sinusoidal waveform and continuously increasing its amplitude while observing the output of the amplifier on an oscilloscope. The amplifier was observed to saturate when the peak-to-peak amplitude of the input waveform was 4.3 ± 0.1 V. The output of the discriminator was then viewed on the oscilloscope as the input waveform amplitude was increased to and beyond the saturation level of the amplifier. The discriminator waveform was observed to vary in such a way that the timing of the FPGA (the timer used in the TeachSpin apparatus) would not be altered. Thus, PMT output signals with amplitudes beyond the saturation level of the amplifier will not distort the data generated by this apparatus.

The behavior of the internal discriminator of the apparatus was then characterized by inputting the same test waveform used previously and observing the output of the discriminator on an oscilloscope while varying the threshold level. The output waveform of the discriminator circuit was observed to be a square wave, while increasing the threshold value was observed to increase the duration of the wave in a continuous fashion while leaving the frequency of the wave unchanged. When the limiting threshold value was reached, the leading and trailing edges of the square waves were observed to meet, resulting in an output signal of constant voltage.

The timing properties of the FPGA were then characterized. First, an input waveform was generated using the internal pulser (a device which simply pulses light) on the detector to simulate muon decays with a lifetime of 6.1 ± 0.2 µs as measured by an oscilloscope. The provided software was also used to analyze this data, and the software returned a decay time of 6.100 ± 0.003 µs, which is in good agreement with the value measured by the oscilloscope.

The linearity of the FPGA was then measured by using the internal pulser on the detector to simulate muon decays with lifetimes between 1 and 20 µs as measured by an oscilloscope. The provided software was also used to analyze these simulated decays, and the lifetime reported by the software was recorded. A plot of this data was generated, and is shown below in Fig. 1.

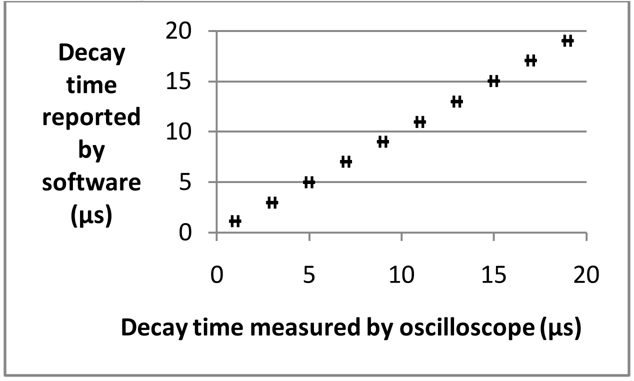

Figure 1—Decay time reported by software vs. Decay time observed on oscilloscope

As can be seen from the plot, the linearity of the FPGA is excellent for all of the simulated decay times measured.

The timeout interval of the FPGA was then measured by increasing the interval length between successive pulses used to trigger the FPGA as measured by an oscilloscope. When the interval length was increased to 19.94 ± 0.16 µs, the FPGA was observed to stop detecting decays. Thus the FPGA will not record decays of muons which take longer than 20 µs to decay after coming to a stop in the scintillator.

The FPGA internal timing bin width was then determined by continuously decreasing the time interval between successive pulses while observing which bin the software placed the decay into. The bin width was observed to be 0.02 ± 0.02 µs. This implies that the minimum observable decay time using this apparatus is roughly 0.01 µs, as decay times between 0 and 0.02 µs will be placed into the first bin, and it is not possible to discriminate amongst them.

The discriminator output rate was then increased by decreasing the threshold value below 1.9 as measured on the dial, which was observed to increase the output rate by a factor of 6. The apparatus was then run without using the pulser in order to obtain actual muon decay data. This resulted in a raw count rate of 72 muons per second, which is well above what was observed when the experiment was run with proper discriminator settings. The muon lifetime as reported by the software was 0.705 ± 0.008 µs, which is well below the accepted lifetime value of approximately 2.2 µs. Thus the minimum threshold value which will yield good experimental results is approximately 2 as measured on the threshold dial.

To determine the optimal high voltage (HV) at which to run the detector, the HV input was continuously varied while the amplifier output was observed using an oscilloscope. The signal used during this calibration was an input signal from the internal pulser on the detector corresponding to muon decays with a lifetime of approximately 2 µs. A setting of less than 5, i.e. 500 V, on the HV dial resulted in no detection of the signal by the detector whatsoever. Between HV settings of 5 and 8.2 the detector registered a muon flux, but did not detect any decays. For HV settings between 8.2 and 10 as read by the dial, both detections and decays were registered, with no significant observable difference when the HV input was varied within this range. When taking real data, the HV setting was thus set to 10.

Measurement of Muon Lifetime Using the TeachSpin Apparatus

The TeachSpin apparatus was set with a HV setting of 10 and a discriminator setting of 6 as read on the dial and left to take data for two weeks. Using this data, a plot of counts vs. decay time was generated and is shown here as Fig. 2.

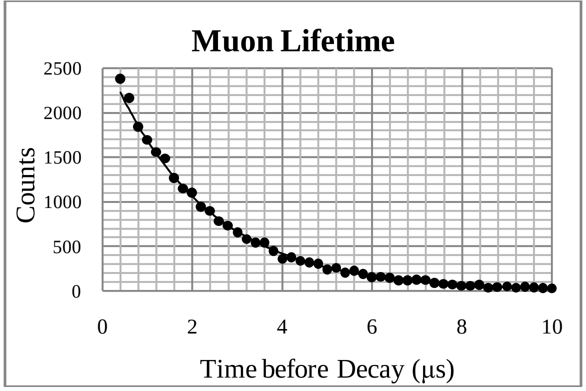

Figure 2—Plot of Counts vs. Time before Decay for TeachSpin Apparatus

For some initial population N_0 of particles decaying at a constant rate $\lambda=1/\tau$, it is well known that the number of particles remaining as a function of time is described by $N(t) = N_0 e^{-\lambda t}$. Differentiating and rearranging this equation one arrives at $-\frac{dN}{N_0} = \lambda e^{-\lambda t} dt$. However, $-\frac{dN}{N_0}$ is just the probability of any single particle decaying. Thus the rate of decay as a function of time is given by $D(t) = \lambda e^{-\lambda t}$, which is an exponential decay with the same parameter λ. This holds for any value of N_0, which in the case of this experiment is 1, as we are only observing discrete decays of single muons which enter the detector at random times. Thus, although the amount of time that the muon has existed before entering the detector is unknown, we can still expect an exponential distribution in the decay times of the muons with the same decay parameter.

The TeachSpin software was used to run a regression and extract the decay parameter λ from the data. As the experimental value for λ is the mean rate of decay per unit time, $\tau = \frac{1}{\lambda}$ is the corresponding experimental mean time before decay, i.e., the lifetime. The software reported a value of $\tau = 2.208 \pm 0.014\ \mu s$, which compares well with the accepted mean muon lifetime of 2.19703 ± 0.00004 μs.[1]

However, while this experimental value reported by the TeachSpin software is in agreement with the mean muon lifetime in free space, due to the existence of a secondary decay mode for negatively charged muons in the scintillator, it is expected that the observed mean lifetime will be slightly shorter than the free space value. Due to the fact that the FPGA is unable to detect muon decays as the decay lifetime approaches zero, some of the data was artificially electronically truncated by the apparatus. To remedy this, these artificially low data points were excluded and another regression was run. This resulted in a value of τ = 2.16 ± 0.2 μs, which is slightly lower than the accepted free space value, as expected.

The Ratio of Positive to Negative Muons in the Detector

The ratio of positively charged muons to negatively charged muons stopping in the scintillator was also calculated using the experimental value obtained for the mean muon lifetime. As positively and negatively charged muons have different mean lifetimes, the average experimentally observed lifetime can be given as

$$<\lambda> = \frac{N^+\lambda^+ + N^-\lambda^-}{N^+ + N^-}$$

where N^+ and N^- are the number of positively and negatively charged muons, and λ^+ and λ^- are their corresponding decay rates. Defining $\rho \equiv \frac{N^+}{N^-}$, the ratio of the number positively to negatively charged muons, this equation can be rewritten as

$$\frac{1}{<\lambda>} = \tau^{obs} = (1+\rho)\left(\frac{1}{\tau^-} + \frac{\rho}{\tau^+}\right)$$

where τ^+ and τ^- are the reciprocals of λ^+ and λ^- respectively, corresponding to the mean lifetimes of positively and negatively charged muons. This can be solved for ρ to obtain the desired

$$\rho = -\frac{\tau^+}{\tau^-}\left(\frac{\tau^- - \tau^{obs}}{\tau^+ - \tau^{obs}}\right)$$

Using $\tau^- = \tau_c$, where $\tau_c = 2.043 \pm 0.003\ \mu s$[2] is the lifetime of negative muons in carbon (corresponding to the scintillator), and $\tau^+ = \tau$ is the free space mean lifetime of muons, as the positively charged muons do not interact with the scintillator nuclei, one arrives at a value for the charge ratio of $\rho = 3 \pm 2$. Comparing this value to a published experimental value of 1.35 ± 0.10, one finds our value is in good agreement.[4] Of course, collecting more data would improve the precision of our measurement.

Calculation of the Fermi Constant

The value of the Fermi coupling constant G_F was also calculated using the experimentally obtained value for the mean lifetime of the muon. As stated above, the coupling constant can be calculated using

$$G_F = \sqrt{\frac{192\pi^3\hbar^7}{m^5c^4\tau}}$$

Using the value of τ as reported by the TeachSpin software and the value $m = 105.658376 \pm 0.000004\ \mathrm{MeV/c^2}$ from the particle data group, one arrives at a value of the Fermi constant of $G_F = (1.429 \pm 0.005) \times 10^{-62}\ \mathrm{J{\cdot}m^3}$, which compares well with the value listed on the particle data group website of $G_F = (1.43584 \pm 0.00001) \times 10^{-62}$ J·m^3.[3]

Comparison of Muon Decay Spectrum with Theoretical Binomial Distribution

A fresh data sample of 2000 decay events was taken, which were subdivided into groups of 50 events. The number of decays which occurred in less than one muon lifetime was recorded for each group. This distribution of decays was then compared to that predicted by the theoretical binomial distribution. As 1-(1/e) = 0.63 is the probability of decay within one lifetime, this value was used as the value of the probability of success, p, for the binomial distribution. Given a sample of size n, theory predicts that the mean number of successes in n trials will be $np = 31.5$. Theory also predicts a variance of $\sigma^2 = np(1-p) = 11.6$. These values were calculated using our experimental data. The experimental mean number of successes was calculated to be 31.6, and the experimental variance was calculated to be 12.3. These experimental values compare favorably with the theoretically predicted values, particularly when one considers the limited sample size used in the experimental calculation.

Determination of the Muon Lifetime Using the Homemade Apparatus

The lifetime of the muon was measured by the homemade detector in exactly the same way as the TeachSpin apparatus. Below in Fig. 3 is the calibrated graph of counts vs. decay time.

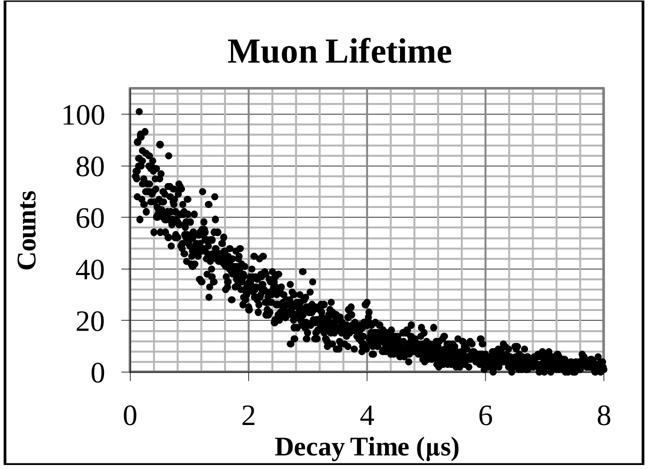

Figure 3—Plot of Counts vs. Decay Time for Homemade Apparatus

In order to calculate the lifetime of the muon, the natural logarithm of the counts were graphed verses the measured decay time. Figure 4 below shows this graph.

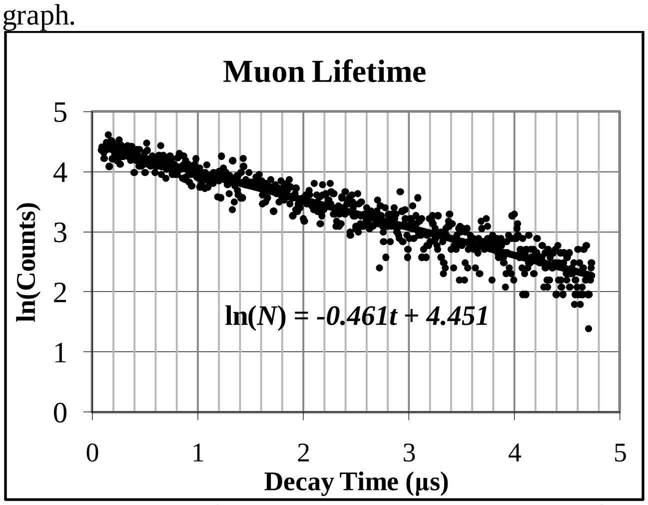

Figure 4—Plot of ln(Counts) vs. Decay Time for Homemade Apparatus

Note that this graph was truncated on the right because the number of counts for very large decay times was very small. The discrete nature of the data caused the graph in Fig. 4 to deviate widely from a line for large values of decay time. If data were taken for a longer time, this truncation could be avoided.

The slope of the graph in Fig. 4 yields a decay time of 2.17 ± 0.03 μs, which agrees well with the accepted value of 2.19703 ± 0.00004 μs.[1]

A Discussion of False Counts

As explained above, false counts result from two incident muons being detected at nearly the same time. In order to calculate how often this occurs, the total number of stop signals in a 10 minute interval was measured as was the number of muons passing through the detector in a 10 minute interval of time. This resulted in rates of $R_{stop} = 7.82 \pm 0.11$ Hz and $R_{start} = 8.53 \pm 0.12$ Hz respectively. The rate of false decay counts is then given by:

$$R_{false} = R_{start} \cdot R_{stop} \cdot T_{range}$$

where T_{range} is the range setting on the TAC, i.e. how close the start and stop signals would need to be in order to register a false count. This equation gives a value of $R_{false} = (6.67 \pm 0.13) \times 10^{-4}$ Hz. This can be compared to the total rate of decays as measured by the apparatus: $R_{tot} = (4.31 \pm 0.03) \times 10^{-2}$ Hz. One thus finds that on average, only about $1.55 \pm 0.03\%$ of the measured decays would be false decays for this apparatus.

Conclusion

The lifetime of the muon was measured twice, yielding values of $\tau = 2.16 \pm 0.02\mu s$ and 2.17 ± 0.03 μs, both of which agree well with the accepted value and fit with the expectation that the lifetime in matter is slightly shorter than the lifetime in free space. For the homemade apparatus, the number of false decays was calculated to be only about 1.6% of all measured decays, thus showing that, while not entirely negligible, the false decays should not significantly affect the data. The ratio of the number of positively charged muons to the number of negatively charged muons was calculated to be 3 ± 2, which is in agreement with published experimental data. The Fermi Coupling Constant was calculated to be $G_F = (1.429 \pm 0.005) \times 10^{-62}$ J·m^3, which compares well with the accepted value of $G_F = (1.43584 \pm 0.00001) \times 10^{-62}$ J·m^3.

References

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3. Particle Data Group Website - http://pdg.lbl.gov
4. Bahmanabadi, M.; Ghomi, M..; Sheidaei, F. "Experimental Studies of Positive and Negative Atmospheric Muons with a Cosmic Rays Telescope", **Astroparticle Physics**, Volume 24, Issue 3, October 2005, Pages 183-190.