Chapter 6

Experimental Sagnac Interferometer

6.1 Experimental Set-Up

The experimental set-up of the Sagnac interferometer is identical to that used to make transmission measurements (Fig. 4.1 on page 76) up to the optical fibre. After the pump and probe beams have left the optical fibre, the set-up is significantly modified to form the Sagnac interferometer, Fig. 5.3 on page 116. The optical fibre ensures that the pump and probe beams co-propagate.

The Sagnac interferometer is formed by the loop originating at the second beam splitter (BS2). Probe beams propagate in both directions around the loop, while the pump beams are coupled out of the loop by the polarizing beam splitting cube; only the clockwise propagating pump beam passes through the Rb vapour cell. After completing the loop of the interferometer the two probe beams interfere on BS2. 50% of the interference propagates towards photodiode A and 50% towards the first beam splitter (BS1). A neutral density filter (ND) ensures that only 25% of the output form the interferometer is incident upon photodiode A. On BS1 the output of the interferometer is divided again such that 25% of the output of the interferometer is incident upon photodiode B.

In practice the beam splitters do not perform as perfect 50 : 50 beam splitters. This leads to a small difference in the obtained signals from those predicted in equations 5.62 and 5.63 on the preceding page. This is overcome by making
small adjustments to the mechanical slits.

In order to ensure that the two arms of the interferometer counter-propagate, a Watec high resolution CCD camera (WAT-902B) with a Computar 25 mm lens was used to monitor the degree of overlap of the beams on the mirrors within the interferometer. An iterative process — monitoring the overlap on mirror A, while adjusting mirror B, then monitoring mirror B whilst adjusting mirror A — allows the beams to be brought very close to counter-propagating. Monitoring the beams in the two output arms of the interferometer allows the counter-propagating arms within the interferometer to be brought to be "perfectly" counter-propagating. When the beams are perfectly counter-propagating and there is no absorbing medium present in the interferometer output arm B will be bright, while arm A will be dark — encapsulated in equation 5.42 and equation 5.39, and shown in Fig. 6.1(upper).

![Figure 6.1](image)

**Figure 6.1:** The output beam profiles for the two arms of the Sagnac interferometer, for the case of perfectly counter-propagating beams and for the biased alignment that leads to the dispersion signals. Arm A profiles are shown on the left and Arm B on the right. The grey rectangles show the position of the mechanical slits.

A change in the refractive index of the medium for one direction of propagation shifts the fringe pattern. However the sensitivity is minimal, as the shift is about a maximum or minimum of the interference pattern, where the rate of change of
Figure 6.2: The output beam profiles of the Sagnac interferometer, as recorded on the Watec CCD camera, for the biased alignment. Arm A is on the left-hand side while arm B is on the right-hand side. (i) Shows the profiles before the slits. (ii) The beam profiles as viewed on the slits. (iii) The beam profiles of the light transmitted through the slits.

intensity with displacement is lowest. To enhance the sensitivity we “bias” the interferometer by introducing a small angle between the counter-propagating beams, [99, 101], such that both light and dark fringes appear in the interference pattern at both outputs, Fig. 6.1(lower) and Fig. 6.2(i). Two mechanical slits aperture the fringe pattern, such that only the region between the light and dark fringe is focussed onto the photodiode, Figs. 6.1(lower), 6.2(ii) and (iii). This biasing technique enables one to obtain maximal sensitivity to changes in the refractive index and a signal that is directly proportional to the refractive index difference between the two counter-propagating probes, § 5.3. For all
traces recorded with both the pump and probe propagating through the vapour cell, traces were also recorded with the pump beam blocked before the Sagnac interferometer. This alignment procedure was adopted so that the probe-only signal could be subtracted from the pump-and-probe signal — hence allowing any features in the scan across the resonance that are not due to the presence of the pump beam to be removed from the spectra.

6.1.1 Photodiode Circuit

In work presented in chapter 4 of this thesis, lock-in amplifiers have been used to detect the EIT resonances. Concern was raised that the lock-in amplifier may be limiting the line shape of the transmission resonances, [104]. In order to avoid the use of a lock-in amplifier, and yet still be able to measure the EIT resonances it was necessary to use a photodiode circuit with lower noise levels than that which had previously been used. The photodiode circuit used throughout this chapter is shown in Fig. 6.3. The main difference between this circuit and those used in experiments in previous work in this thesis, is that in this case the photodiode is not biased. This leads to two main advantages over the previous circuits, a lesser influence from dark currents, and a wider range over which the photocurrent is linear with radiant intensity, [40, 41, 105]. The improvement in signal to noise ratio of this photodiode circuit, over that used in earlier work in this thesis, was investigated by J. Gaffney1.

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1This work was carried out during a Nuffield funded summer project.
The output voltage of this photodiode circuit has the same form of temporal filtering as the photodiode circuits used in chapter 4, shown in Fig. 4.9 on page 86. However with a different value of $R = 10 \text{ M}\Omega$, the output voltage will be a factor of 10 higher and will have a frequency cut-off as shown in Fig. 6.4. With the impedance decreasing significantly over angular frequencies $\simeq 10$ kHz, it follows that changes in signal on a time scale $\ll 100 \mu$s will be heavily filtered.

![Figure 6.4: The output voltage of the photodiode circuit is directly proportional to the output impedance. The calculated output impedance of the photodio
are shown in Fig. 6.5. Both pump and probe beams have circularly-symmetric Gaussian profiles, as expected from the circular core of the optical fibre, [106], in agreement with perpendicular sets of beam profile measurements. The anticlockwise beams travel a distance of 2.2 m from the output of the optical fibre to the slit, whereas the clockwise beams propagate 0.7 m from the fibre output to the slit. The clockwise pump and probe fields have a 1/e full-width of 1.890 ± 0.007 mm. The anticlockwise probe beam has a 1/e full-width of 3.177 ± 0.003 mm. The difference in beam size is as expected due to the different path lengths of the two beams around the interferometer to the point where the beam profile was measured.

Figure 6.5: The red plots show a typical beam profile for the clockwise beams: experimental measurements (circles) and Gaussian fit (line) with a 1/e full-width of (1.890 ± 0.007) mm. The blue plots show a typical beam profile for the anticlockwise beams: experimental measurements (circles) and Gaussian fit (line) with a 1/e full-width of (3.177 ± 0.003) mm.
6.3 Double Scan

As in the work presented in § 4.2, both the pump and probe beams are derived from the same Extended Cavity Diode Laser (ECDL) and double-pass through separate Acousto-Optic Modulators. The pump and probe have orthogonal-circular polarizations. The ECDL is scanned about \( \delta_{pu} = 0 \) and the probe AOM is scanned about \( \delta_{pr} - \delta_{pu} = 0 \), Fig. 4.14 on page 91. Fig. 4.15 on page 92(i) shows the control voltages to the ECDL piezo and the probe VCO. Fig. 4.15 on page 92(ii) shows the plot of \((\delta_{pr} - \delta_{pu})/2\pi\) against \(\delta_{pu}/2\pi\). The

![Figure 6.6](image)

**Figure 6.6:** (i) and (ii) are the probe-only output signals of arm A and B respectively. (iii) and (iv) are the probe-and-pump output signals of arms A and B respectively. The plots show both the raw photodiode signals (red) and a Gaussian fit to the signal (blue). The power of the clockwise pump and probe beams are 26 \(\mu\)W and 4.2 \(\mu\)W.

interferometer outputs are plotted in Fig. 6.6, along with Gaussian least-square
fits of the form,

\[ A\nu + B - Ce^{-\left(\frac{\nu - \nu_0}{w}\right)^2}. \] (6.1)

\( A \) and \( B \) provide the fit to the off-resonance transmission background. \( A \) is the gradient of the linear offset, \( B \) is the zero frequency offset, \( C \) is the amplitude of the Gaussian absorption, \( \nu \) is the frequency of the light, \( \nu_0 \) is the frequency of the centre of the Gaussian and \( w \) is the 1/e full-width of the Gaussian. The Gaussian fit is subtracted from the photodiode signal for each of the four traces shown in Fig. 6.6 on the previous page. The probe-only signal is subtracted from the pump-and-probe signal, the resulting traces show only the non-linear features, with the Gaussian backgrounds subtracted, see Fig. 6.7 on the following page.

The signals are normalized by dividing them by the off-resonance probe-only sum signal. For the purpose of this normalization the off-resonance signal is taken to be \( A\nu_0 + B \) minus the recorded signal for both pump and probe beams blocked.

Summing the normalized signal for arm A and arm B leads to the signal proportional to the absorption. This is shown in Fig. 6.7(i). The difference signal between arms A and B is shown in Fig. 6.7(ii). This is proportional to the dispersion.

The double-scanning technique leads to \( m \) EIT features occurring within the range of the Doppler-broadened transition, Fig. 6.7 on the next page. The frequency scale of such a plot is not straight forward since the centres of the different EIT features are separated by a frequency given by the ECDL scan, \( \delta_{pu} \), whilst the width of the individual features is determined by the AOM scan, \( (\delta_{pr} - \delta_{pu}) \).
Figure 6.7: Difference between pump-and-probe and probe-only signals with the Gaussian fits subtracted, for output arm A, (i), and output arm B (ii). The residual signals plotted above show the modification in the two output arms due to the two-photon resonance condition being met.

As can be seen in Fig. 6.8, there is still a residual background. In order to be able to characterize the variation of amplitude of EIT feature with single-photon detuning, this background has to be removed. This is done by fitting a Gaussian envelope to the off-two-photon resonance background, Fig. 6.9(i). This fit is subtracted from the signal and a Gaussian function can then be fitted to the amplitudes of the EIT features, Fig. 6.9(ii).

Scanning two counter-propagating beams at the same frequency across the Doppler-broadened resonance leads to the occurrence of saturation spectroscopy resonances, [27, 37]. The features most prominent in Fig. 6.9(ii) occur at frequencies of approximately 0 MHz, −80 MHz and −160 MHz, corresponding to $F = 1 \rightarrow F' = 2$, $F = 1 \rightarrow F' = 1, 2$ cross-over resonance, and $F = 1 \rightarrow F' = 1$ respectively.
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Figure 6.8: (i) Sum of arm A and arm B signals as presented in Fig. 6.7. This shows the array of EIT transmission signals due to the two-photon resonance condition being met at a number of different single-photon detunings. (ii) Difference between arm A and arm B signals as presented in Fig. 6.7. This plot shows the array of dispersive features due to the EIT two-resonance condition being met at a number of different single-photon detunings. The unsmoothed data is shown (red) along with a twenty-point moving average (blue). The twenty-point moving average involves taking the mean of twenty consecutive data points and plotting this mean value at the centre frequency of the set of twenty data points.
Figure 6.9: (i) moving average of the sum signal from Fig. 6.8(i) (red), with a least square Gaussian fit to the background (blue). (ii) Sum signal data minus the Gaussian fit to the background (red) is shown with a Gaussian fit to the amplitude of the EIT transmission features (blue). This is to show that the variation in amplitude of the EIT features as a function of single-photon detuning, can be represented by a single Gaussian function, with width of the same order as the Doppler-broadened resonance.
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The amplitude of the EIT signals are determined by a Gaussian envelope of FWHM 560 MHz, this compares to the FWHM of 620 MHz of the Gaussian fit to the Doppler-broadened resonance. The uncertainty on the fit to the peaks of the transmission signals is likely to be far higher than the uncertainty on the fit to the Doppler-broadened resonances. While it is expected that the two Gaussians would be similar it is not the case that they should be the same. The Doppler-broadened absorption is the sum of three Gaussians, § 2.4 on page 25, each of which corresponds to a different hyperfine transition, which in turn will contribute a different amount to the EIT signals, though exactly to what extent this is the case is beyond the scope of this PhD.

For the intensities of the fields used in this case the maximum amplitude of the transmission EIT signals is 2% of the transmitted light.

6.4 Single Scan

In order to characterize both the transmission and dispersion of the EIT feature, the ECDL was tuned to the frequency at which the amplitude of the EIT features is at its maximum, between −100 and −200 MHz in Fig. 6.9. The probe alone was then scanned across the two-photon resonance. The signals from the two output arms, A and B, were recorded, both with and without the pump field. Plots of the two individual arms of a typical signal can be seen in Fig. 6.10(i) arm A and (ii) arm B, both with (red) and without pump beams (blue).

The frequency scale of Fig. 6.10 (and of subsequent spectra shown in this chapter) is given by \((\delta_{pr} - \delta_{pu})/2\pi\), the detuning of the probe beam from the two-photon resonance. Note the presence of the beat note in the pump-and-probe traces at \(\approx 1.6\) MHz. The traces shown in Fig. 6.10, all have arbitrary DC offsets. The DC offsets are provided using the DC Bias Boxes Fig. 6.11 and Fig. 6.12. The bias boxes allow for a DC offset of between ±9 V to be added to the photodiode signal. This allows the voltage scale on the oscilloscope to be set to maximise the resolution of the EIT signals, whilst still being able to record all of the data of interest. For each data set recorded with the arbitrary DC offset, scans across the full Doppler-broadened transitions were also recorded so
that the data could be normalized if required.

The probe-only signal is subtracted from the pump-and-probe signal. The resulting traces are shown in Fig. 6.13 (i) and (ii), arms A and B respectively. Along with the EIT traces from the two output arms (red), are linear least square fits to the off-resonant background signal (blue). The linear fit is subtracted from the EIT signal in each case, and the remaining signal is normalized, in the same way as in § 6.3 on page 128. The resulting normalized traces for arms A and B are plotted in Fig. 6.13 (iii) and (iv) respectively.
Figure 6.10: (i) and (ii) show the raw probe-only (blue) and raw probe-with-pump (red) signals, for output arms A and B respectively. The clockwise pump and probe powers are 28 µW and 3 µW, respectively.
Figure 6.11: Switch S1 turns the DC bias on or off. Switch S2 can be in either of two positions, position 1 (as shown in the figure) or position 2. This determines whether the bias is positive or negative, Fig. 6.12. \( R \) is a 5 kΩ potentiometer, adjusting this potentiometer varies the magnitude of the DC offset from 0 V to 9 V.

Figure 6.12: (i) Switch S2 in position 1 as shown in Fig. 6.11 gives a negative DC offset. (ii) Switch S2 in position 2, leading to a positive DC offset on the output.
Figure 6.13: (i) and (ii) show the difference between the pump-and-probe and probe-only signals (red) along with a linear fit to the background (blue), for output arms A and B respectively. (iii) and (iv) show the normalized traces for arms A and B respectively.
The two normalized traces of Fig. 6.13 (iii) and (iv) are summed to give the signal proportional to the EIT transmission signal, Fig. 6.14(i). The difference signal, proportional to the difference in refractive index between the two arms, and hence proportional to the dispersion due to the EIT resonance, is plotted in Fig. 6.14(ii).

![Normalized Signal vs Frequency](image)

**Figure 6.14:** (i) Sum signal of output arms A and B, with (blue) and without (red) twenty-point running average, showing a typical EIT transmission signal. (ii) Difference signal between output arms A and B, showing a signal proportional to the dispersion of the medium around the EIT two-photon resonance.

### 6.5 Sagnac Interferometer Output Beam Alignment

With the counter-propagating arms of the Sagnac interferometer aligned as described at the beginning of this chapter, in the “biased” alignment, no aperturing of the output beams leads to a difference signal as shown in Fig. 6.15.
Fig. 6.16 shows the difference signals for the aperture positions shown in Fig. 6.17,

\[
\begin{array}{cccccccc}
-300 & -150 & 0 & 150 & 300 \\
-8 & -4 & 0 & 4 & 8 \\
\end{array}
\]

**Figure 6.15:** Difference signal recorded for no slits in the output arm fringe patterns. The clockwise probe power is 3 $\mu$W and the pump power is 10 $\mu$W. As is clearly shown in the figure, the spectrum in the absence of the output arm apertures is not of the form of a dispersion spectrum.

(i) and (ii) have the aperture positions in each of the two output arms being on the same side of the beams, while (iii) and (iv) have the apertures on opposite sides of the output beams.
Figure 6.16: (i) Standard aperture position. (ii) Both slits on the opposite side of beams. (i) and (ii) show that provided both slits are set to the correct width and that they are both on the same sides of the two beams, a signal proportional to the dispersion of the medium will be obtained. (iii) Slit A on standard side of beam, slit B on opposite side of beam. (iv) Slit A on opposite side of beam, slit B on standard side of beam. (iii) and (iv) show that if the slits are on opposite sides of the two different beams, the signals will not be proportional to the dispersion. The clockwise probe and pump powers are 2 µW and 10 µW.
Figure 6.17: The aperture positions shown in this figure are those used to record the traces in Fig. 6.16. Each of the four different combinations shown in this figure corresponds to the same plot in Fig. 6.16.
6.6 EIT Line Width

Details of theoretical line shapes are given in chapter 3. If the signals are

![Figure 6.18](image)

**Figure 6.18:** This figure compares three different predicted line shapes to the measured EIT transmission signals. In (i), (ii) and (iii), the experimental data is plotted (red), with the theoretical line shape fit (blue). (i) Lorentzian fit; (ii) arctan fit; and (iii) cusp fit. (iv) Experimental data minus Lorentzian fit; (v) experimental data minus arctan fit; and (vi) experimental data minus cusp fit. The clockwise probe power is 3 µW and the pump power is 28 µW. Power broadened, the beam profile affects the line shape and width of the resonance, [68]: a step-like beam profile leads to a Lorentzian line shape and a Gauss-
sian beam profile to an arctan line shape. The cusp function gives the expected line shape for transit-time dominated broadening, [66], and is virtually indistinguishable from the arctan fit, [68]. Fig. 6.18(i), (ii) and (iii) all show the twenty-point moving average transmission EIT feature with weighted least square fits of the Lorentzian, arctan and cusp functions respectively. Fig. 6.18(iv), (v), and (vi) show the residual signal — data minus theoretical fit — for Lorentzian, arctan and cusp functions respectively. It can be seen that away from the centre of the two-photon resonance (detuning greater than ±200 kHz), the residuals are essentially the same for the three different fits. Only in the central region (detuning less than ±200 kHz) is there any significant difference between any of the residual traces. The arctan and cusp fits are very similar. Either side of the centre of the resonance, the amplitude of the experimental line shape is larger than that of the fit, but on the centre of the resonance (detuning less than ±50 kHz), the fit value exceeds the data by approximately 10%. The Lorentzian fit has a higher value than the data either side of the resonance, but in the region around the centre of the resonance (detuning less than ±30 kHz), the fit is approximately 5% less than the experimental data.

The reduced $\chi^2$ values for each of the three functions suggest that none of the models truly fit the data. Across the two-photon detuning of −300 kHz to +300 kHz the reduced $\chi^2$ values are 5.59, 9.26, and 5.63, for Lorentzian, arctan, and cusp fits respectively. Over the wider detuning of ±900 kHz, the reduced $\chi^2$ values are 6.29, 7.15, and 5.74. Hence away from resonance the cusp fit gives the better fit whilst in the region of the resonance the Lorentzian fit is marginally the best fit. Were any of the functions an accurate fit, then a reduced $\chi^2$ value of between one and two would be expected. In order to quantify the FWHM and amplitude of the EIT resonances for different pump and probe powers, the Lorentzian model was used. Rather than fitting the Lorentzian to the sum signal, the Lorentzian dispersion is fitted to the difference signal between the two arms. The Lorentzian dispersion line shape takes the form of equation 3.80 on page 70,

$$n_R - 1 = \frac{Nd_{ba}^2}{2\varepsilon_0 \hbar} \cdot \frac{\delta_{pr} - \delta_{pu}}{(\delta_{pr} - \delta_{pu})^2 + (\Gamma_a/2)^2}.$$
6.6.1 EIT Line Shape Dependance on Pump and Probe Power

EIT resonance spectra are recorded for three different regimes of pump and probe power, $P_{pu}$ and $P_{pr}$ respectively.

- $P_{pr}$ fixed and $P_{pu}$ varied in the region, $2P_{pr} \lesssim P_{pu} \lesssim 20P_{pr}$.
- $P_{pu}$ fixed and $P_{pr}$ varied in the region, $P_{pu}/40 \lesssim P_{pr} \lesssim 0.8P_{pu}$.
- $P_{pr} = P_{pu}$, and both are varied.

**Dependence on Pump Power**

Variation in amplitude and FWHM of the two-photon EIT resonance, with increasing pump beam power, is plotted in Fig. 6.19(i) and (ii) respectively. The amplitude of the features increases with pump power until it starts to saturate at around 20 $\mu$W, Fig. 6.19(i).

![Figure 6.19](image-url)
Reducing the intensity of the pump reduces the width of the resonances, as can be seen in Fig. 6.19(ii). Extrapolating the linear fit of Fig. 6.19(ii), shows that reducing the pump power to zero will lead to a FWHM of 170 kHz, where the probe power is 4 µW.

**Dependence on Probe Power**

![Graph showing the relationship between probe power and FWHM](image)

**Figure 6.20:** (i) Plot of the amplitude of the EIT signal against probe power. (ii) Plot of the FWHM of a Lorentzian dispersion fit to the difference signal for a range of pump powers. The pump power is 10 µW.

Fig. 6.20(i) shows that the amplitude of the EIT signal increases linearly with probe power (with a constant offset). This is to be expected in the case that the probe does not effect the amplitude of the absorption coefficient. The linear increase in the measured amplitude is due to the linear increase in the power of the incident beam. There is one notable exception, the data point at \( \simeq 6.5 \) µW falls noticeably below the linear line of best fit. The most likely explanation for this is that the detuning of the pump beam, \( \delta_{pu} \), had drifted from the centre of the Doppler-broadened resonance. This would result in a reduction of the EIT resonance amplitude, (Fig. 6.9 on page 132).
Chapter 6. Experimental Sagnac Interferometer

Fig. 6.20(ii) shows the variation of FWHM of the EIT features with probe power. As can be seen the width of the resonance increases approximately linearly with probe power hence this data is not in the regime of a weak probe. In the weak probe regime the probe power would not effect the EIT resonance. This is to be expected, as with a pump power of 10 μW, the probe powers of ~ 1 μW to 8 μW are a significant fraction of the pump power, and as such can not be described as being in the weak probe regime.

Dependence on Simultaneous Variation of Probe & Pump Power

![Graph showing variation of FWHM and amplitude with probe power](image)

**Figure 6.21:** (i) Plots of amplitude of EIT signals against the beam power. (ii) Plots of the FWHM of the EIT resonances against the beam power.

By definition when the pump power is equal to the probe power, the weak probe regime cannot apply.

Fig. 6.21(i) shows that the amplitude of the feature increases at a rate greater than linear. There will be two mechanisms leading to the increase in amplitude. Increasing probe power will lead to a linear increase in amplitude, as seen in Fig. 6.20(i). Secondly increasing pump power will lead to an increase in the
transmission of the medium, Fig. 6.19(i). As the probe and pump power are equal then both will contribute equally to the “pumping” of the medium.

Fig. 6.21(ii) shows the variation in FWHM with beam power. It is apparent that the data point corresponding to a beam power of $\approx 0.5 \mu$W is not plotted. This is due to the fact that the signal to noise ratio was such that whilst the amplitude could be fitted to an acceptable degree of precision, the FWHM could not. The data suggests that the increase in FWHM is at an order greater than linear.

6.6.2 Comparison of EIT Lineshape to Theory

For the system under investigation, $^{87}$Rb $^2S_1/2 \ F = 1 \to 5 \ ^2P_{3/2} \ F' = 0, 1, 2$, the Doppler-broadened resonances have a significant overlap, Fig. 2.4 on page 28. Thus it is not possible to consider the experimental EIT resonances as being due to a single $\Lambda$ system. The resonances are due to three different $\Lambda$ systems, with the lower states $|F = 1; m_F = \pm 1 \rangle$ and excited states, $|F' = 0, 1, 2; m_{F'} = 0 \rangle$.

The single photon transitions in each of the three $\Lambda$ systems have different transition strengths. Thus for the same beam intensities the Rabi frequencies differ by the ratio of the square root of the transition strengths, the ratio of the dipole matrix elements. The dipole matrix element ratio is $2 : \sqrt{5} : 1$, for $F' = 0, 1, 2$ respectively$^2$, [32]. Across the full range of the Doppler-broadened resonance, the amplitude of the contribution of each different $\Lambda$ system varies with frequency due to the frequency separation between the excited hyperfine states and due to the Rabi frequency varying with detuning.

Thus it is not trivial to account for the line widths of the measured EIT resonances using the theoretical models presented in chapter 3.

Magnetic Broadening

An axial magnetic field of $\approx 1$ G is applied to shift the EIT resonances away from the beat note. This field will lead to broadening of the EIT resonance as

$^2$On this scale the closed transition, $|F = 2; m_F = +2 \rangle \to |F' = 3; m_{F'} = +3 \rangle$ has a transition strength of $2\sqrt{3}$. 
the field is not constant along the length of the vapour cell. From § 4.3.4 on page 102, the broadening of the resonances can be approximated by \( \simeq 3 \text{ kHz G}^{-1} \).

Hence the contribution, of magnetic broadening, to the width of the EIT resonances presented in this chapter will be \( \simeq 3 \text{ kHz} \).

**Transit Time**

The cusp line shape predicted for transit time broadening only applies in the regime where the pump and probe power do not contribute to the broadening of the resonance, § 3.5.4 on page 62. As can clearly be seen from Figs. 6.19, 6.20 and 6.21, the pump and probe powers do contribute to the width of the resonances. Hence the expected line shape is not that predicted in § 3.5.4.

However it will still be instructive to consider the contribution of transit time broadening to the measured line widths. From equation 3.51,

\[
\Gamma_{\text{EIT}} = \frac{\sqrt{2} \ln 2 \nu}{r}.
\]

\( r \) being the intensity \( 1/e \) beam radius, and \( \nu \) the velocity given by equation 3.52 on page 63. Thus \( \Gamma_{\text{EIT}} = 39.1 \times 2\pi \text{ kHz} \), at room temperature, 293 K, for \( ^87\text{Rb} \).

**Beam Profile**

In order for the line shape to be determined by the pump and probe beam profiles, the relaxation rate of the atoms, \( \Gamma_L \), must be much less than the inverse of the transit time of the atoms through the beam, equation 3.55.

However the relaxation rate of the atoms is limited by the transit time of the atoms though the cell. The transit time through the cell is not significantly greater than the transit time of the atoms through the beams. Hence the measurements presented in this chapter do not fall into the regime where the beam profile determines the line shape.

**Doppler-Broadened Limit**

In the case that a single-photon resonance is Doppler broadened, it follows that an EIT resonance can be narrowed by the same Doppler-broadening mechanism,
§ 3.5.6 on page 65.

In the presence of Doppler broadening, the FWHM of the EIT resonance, $\Gamma_{\text{EIT}}$, is given by equation 3.68 on page 65,

$$\Gamma_{\text{EIT}}^2 = \frac{\gamma_{cb} \Omega_{pu}^2}{\Gamma_a} \cdot (1 + x) \left( 1 + \frac{4x}{(1 + x)^2} \right).$$

$x$ is the dimensionless variable given by equation 3.69 on page 65,

$$x = \frac{\Gamma_a}{2\gamma_{cb}} \cdot \left( \frac{\Omega_{pu}}{W_D} \right)^2.$$

The Doppler-broadened width can be taken from table 2.2 on page 37, hence, $W_D = 570 \times 2\pi$ MHz and $\Gamma_a = 6.065 \times 2\pi$ MHz.

The ground state coherence decay rate could be limited by any of the following three mechanisms:

- collisions of atoms with cell walls;
- collisions of atoms with other atoms;
- and atoms leaving the beam.

As the cell diameter is $\sim 10$ times larger than the beam diameter, and the cell length is 4 times the cell diameter, then the rate at which atoms will leave the beams will dominate over the rate at which they collide with cell walls.

The mean free path of the atoms, $l$, is the mean distance they travel between collisions with other atoms. This is given by,

$$l = \frac{1}{\sigma N}, \quad (6.2)$$

$N$ is the total number density, and $\sigma$ is the cross section for Rb—Rb collisions. At room temperature (293 K), $N = 5.65 \times 10^{15}$ m$^{-3}$, equation 2.101, and $\sigma \simeq 2 \times 10^{-18}$ m$^{-2}$, [107], hence the mean free path, $l \simeq 90$ m. As $l$ is $\sim 5000$ times larger than the diameter of the the cell, this will not limit $\gamma_{cb}$.

Thus the dominant mechanism in determining the ground state coherence decay rate, $\gamma_{cb}$ is transit time of the atoms through the beam. Hence,

$$\gamma_{cb} \simeq \frac{r}{v}, \quad (6.3)$$

where $v = \sqrt{\frac{2k_B T}{m}}$. 

Thus, $\gamma_{cb} = 39.6 \times 2\pi$ kHz at room temperature, 293 K, and with the beam diameter 1.89 mm, from § 6.2.1.

The Rabi frequencies for the three different transitions, $\Omega_{F'=0}$, $\Omega_{F'=1}$, and $\Omega_{F'=2}$ are given by,

$$\Omega_{F'=0} = \frac{1}{\sqrt{3}} \cdot \Omega,$$  

$$\Omega_{F'=1} = \sqrt{\frac{5}{12}} \cdot \Omega,$$  

$$\Omega_{F'=2} = \frac{1}{\sqrt{12}} \cdot \Omega,$$

where $\Omega$ is the Rabi frequency for the closed transition on the $^{87}$Rb D$_2$ line, given by equation 2.59 on page 19,

$$\Omega = \frac{\Gamma_a}{\sqrt{2}} \cdot \sqrt{\frac{I}{I_{\text{sat}}}}.$$

In order to make predictions of $\Gamma_{\text{EIT}}$ the intensity, $I$, is taken to be the mean intensity over an area encompassing a fraction $Z$ of the power, $P$, in the beam, § H.2. Therefore the intensity is taken to be, $I_Z$ by equation H.16,

$$I_Z = \frac{Z P}{\pi r_0^2 \ln \left| \frac{1}{1-Z} \right|},$$

where $r_0$ is the 1/e intensity radius.

With all of the above it is now possible to calculate $\Gamma_{\text{EIT}}$ for the three different $\Lambda$ systems. Plots of $\Gamma_{\text{EIT}}$ against pump power up to 100 $\mu$W are shown in Fig. 6.22, where $Z$ is taken to be 0.95.

In the case of the regime shown in Fig. 6.22, where $x \ll 1$, the width of the EIT resonance can be well approximated by equation 3.73 on page 66,

$$\Gamma_{\text{EIT}} = \Omega \sqrt{\frac{\gamma_{cb}}{\Gamma_a}}.$$

As $\gamma_{cb}$ and $\Omega$ can only be approximated then there is scope for a systematic error in the predictions presented in Fig. 6.22.

**Resultant Line Width**

The development of a theoretical model, accurately taking account of all broadening mechanisms and optical pumping, is beyond the scope of this experimental
Figure 6.22: The solid red line shows the prediction for the $\Lambda$ system with upper state $|F' = 0, m_{F'} = 0\rangle$, the dashed blue line shows the prediction for the $\Lambda$ system with upper state $|F' = 1, m_{F'} = 0\rangle$, and the dotted green line shows the prediction for the $\Lambda$ system with upper state $|F' = 2, m_{F'} = 0\rangle$.

thesis. In order to make an approximation to the resultant expected line width, due to the contributions of all of the broadening mechanisms presented so far, the widths due to each contribution have been summed$^3$. Here the probe beam is assumed to make the same contribution to the width that a pump beam of the same power would.

The resultant widths are plotted in Fig. 6.23 alongside the experimental measurements for a constant probe power (of 4 $\mu$W) and varying pump, as plotted in Fig. 6.19.

The theoretical curves underestimate the extent of the broadening for pump powers up to $\sim 20$ $\mu$W. For pump powers in the range of $\sim 20$ $\mu$W to $\sim 80$ $\mu$W the rate of change of width and the absolute values appear to be in good agreement with the theoretical values.

A potential explanation for the discrepancy in theoretical and experimental line widths up to pump powers of $\sim 20$ $\mu$W would be that the constant contributions to the line width have been underestimated. In order for this to be the case and yet still to have agreement at higher pump powers, it would follow that the rate of increase in width attributable to the pump power has been overestimated.

Further experiments varying the beam diameters would be instructive in re-

$^3$The convolution of two Lorentzian functions of width $\Gamma_1$ and $\Gamma_2$ is a Lorentzian function of width $\Gamma_3 = \Gamma_1 + \Gamma_2$. 

Figure 6.23: The solid red line shows the prediction for the Λ system with upper state $|F' = 0, m_{F'} = 0\rangle$, the dashed blue line shows the prediction for the Λ system with upper state $|F' = 1, m_{F'} = 0\rangle$, and the dotted green line shows the prediction for the Λ system with upper state $|F' = 2, m_{F'} = 0\rangle$. The pump broadening is taken to be that shown in Fig. 6.22. The probe, magnetic and transit time broadening contributions are those described in the text. The experimentally measured values along with the standard error are plotted with the black data points.

solving this discrepancy. This would allow for investigation of the transit time effect as well as further investigation of the intensity of the pump and probe fields.

6.6.3 Dependance on Beam Diameter

Ideally EIT resonances would have been recorded in the Sagnac interferometer for a range of beam diameters. This would have allowed experimental evaluation of the effect of varying the transit time on the EIT line width.

Preliminary measurements were made of EIT resonances, in transmission only, where a reduction in line width by a factor of 2.2 was seen for a magnification of the probe and pump beams of 3.1. These measurements were made keeping all other variables constant.

Maintaining constant beam power but increasing the diameter does lead to a reduction in intensity. However, comparison of the reduction in intensity, by a factor of 9.6 ($= 3.1^2$), with any of the experimental plots of line width against power (Figs. 6.19, 6.20 and 6.21) show that this is not sufficient to lead to a
reduction in EIT resonance FWHM of 2.2. Magnifying the beams by a factor of 3.1 also has the effect of increasing the transit time of the atoms through the beam by the same factor. Thus transit time broadening would be expected to be reduced by the factor of 3.1.

It should be noted that these measurements were made without the benefit of an optical fibre in the experimental set up and as a result the beam profiles will have been far from Gaussian, due to the numerous optical elements in the beam path.

**Beam diameter in Sagnac interferometer**

Several telescopes, of different magnification, were introduced to the Sagnac interferometer. The aberrations to the wave fronts of the beams due to the lenses were such that, the interference patterns at the output of the interferometer were not well enough defined to allow measurements of the dispersion to be made.

### 6.6.4 Group Velocity

From equation 3.98 on page 72 the group velocity can be calculated at the frequency of the two-photon resonance. To do this the FWHM of the two-photon resonance, the normalized transmission in the absence of the EIT signal, the normalized signal of the peak of the EIT signal and the length of Rb vapour cell must be known. In the absence of the EIT signal, the normalized transmitted signal is 0.9. Taking the data presented in Fig. 6.18, from the Lorentzian fit the FWHM is 230 kHz. The amplitude of the EIT sum signal is $18.5 \times 10^{-3}$ and from equation 5.62 on page 121, assuming that the clockwise and anticlockwise signals are the same size in the absence of the EIT feature, it follows that at the centre of the resonance, the transmission EIT signal is $0.90 + 2 \times 18.5 \times 10^{-3} = 0.937$. This gives a group velocity of $c/650$. 