In The Proper Lifetime of the Muon

Object. To measure the mean life of cosmic ray muons. This experiment will give you exposure to fast signal processing, simple amplifiers and oscilloscope operation. You will learn about using a photomultiplier for charged particle detection and ‘coincidence’ methods for event detection that are standard techniques in particle physics. In addition this lab will teach you about handling massive amounts of data. You may have over 10,000 muon decays to analyze that are the result of many more (millions) independent event triggers.

I. Introduction

The muon is an elementary particle and one of the fundamental constituents of matter. Muons are very similar to electrons, but the muon has about 200 times more mass than an electron. Muon decay is governed by the weak interaction.

The muon is one of the main constituents of cosmic radiation that reaches the surface of the earth. The total observable cosmic ray intensity peaks at an atmospheric height of about 12 km, declining at higher altitudes.

The rate of cosmic radiation on the surface of the earth is of the order of 1/(cm²-minute). A large fraction of this rate is due to muons produced during the decay of charged pions (+ or -, with a small dominance of + by charge conservation because protons are +), themselves produced in the upper atmosphere mainly by galactic protons (also alpha particles and neutrons, with a few heavier nuclei). The muons eventually decay into electrons and neutrinos and it is the rate of this decay which is the object of the present experiment. Both μ+ and μ- will be detected in this experiment the equipment cannot distinguish between them. In vacuum the two lifetimes are expected to be the same, since the μ+ and μ- are antiparticles of each other. A muon neutrino is produced both when the pion decays and produces a muon, and also when the muon decays and produces an electron. In contrast, an electron neutrino is produced only by muon decay. Therefore the ratio of muon to electron neutrinos is about 2:1.

In order to measure the decay constant for a muon at rest (or the corresponding mean-life)

1Negative muons can form atomic states, with orbits much closer to the attracting positive nucleus. von Baeyer and Leiter derive a corresponding lifetime correction ratio (free muon to bound muon) of the form

\[ R = 1 - 0.5(Z\alpha)^2 - 0.06(Z\alpha)^2(m_\mu/M_{\text{nuclear}}) \]

taking into account the muon orbital velocity (relativistically time-dilating the bound lifetime) and two other effects which tend to cancel. The reliability of the recoil (mass ratio dependent) term is uncertain for \( Z > 1 \), but the form indicates that it can be neglected within the typical statistical uncertainty of this experiment, leaving the first correction term.
one must stop and detect a muon, wait for and detect its decay products, and measure the time interval between capture and decay. Since muons decaying at rest are selected, it is the proper lifetime that is measured. Lifetimes of muons in flight are time-dilated (velocity dependent), and can be much longer for $\beta = v/c \approx 1$. ($v =$muon speed, $c =$ speed of light)

- Do a literature search to familiarize yourself with the theory of the muon decay. You may also consult one of the references at the end of this writeup.

II. Apparatus

The figure shows a photograph of the experimental set-up. The detector is the black aluminum cylinder, which has been placed in the wooden pedestal for convenience

Read the TeachSpin lab manual. It is reasonably complete and well written. After reading the TeachSpin manual proceed to the sections below.

Notes before starting:

- To get sufficient statistics for proper data analysis, you will need to acquire data for several days.
- From the Desktop of the computer, the data taking software is “Muon Decay/Windows/muon_data/muon.exe.”
- The pulser switch on top of the detector controls an LED diode which simulates muon events in the photomultiplier at a rate of 100Hz. This function is useful for initial set up of the data collection parameters and for triggering the scope. Make sure to TURN OFF the pulser during the actual muon data collection. For this part you will want to consult the Error analysis of decays in the supplementary information on our website.
To collect data you will need to select COM4 in the Configure panel of the muon data collection program. You also need to set a histogram scale and bin number. Use your knowledge of the decay time and the expected functional dependence to set the values for these parameters that will produce the best quality data.

The FPGA is the timing circuit of the electronics box. It acts as a gate: the “gate” opens when a muon enters the detector and closes within a predetermined time interval. If there is a decay event during this time interval the software registers a decay “event” which appears on the screen. The only access to this function is through the mouon.exe program. Whenever asked to do an exercise involving the FPGA you will need to run the muon program and monitor for example the Rate Meter.

When connecting a signal to the scope (amplifier or discriminator signal) you will need to terminate the input with a BNC 50Ω terminator in order to prevent distortions caused by reflections due to impedance mismatch as shown in the image below.

Reducing reflection by 50Ω termination at scope input. The termination uses a T and a 50Ω resistor to ground.

**Experimental Procedure**

Additional equipment you will use in this experiment:

- Function generator
- Oscilloscope [read oscilloscope manual](#) on course website. Note: Connection between the amplifier or discriminator and the scope should always be terminated through a 50Ω terminator (use a T-connector).
- Multimeter

**Initial Setup**

1) Connect the power cable and connect the PMT detector output to the PMT input of the Muon Physics box (see figures 4 and 5 in the teachspin manual). Connect the communication cable between the back of the electronics box and the USB connection on the PC.
2) Turn on the power to the electronics box - the switch is at rear of the box. The red LED power light should now be shining steadily. The green LED may or may not be flashing.
3) Set the high voltage (HV) between -1100 volt and -1200 volt using the dial at the top of the detector tube (see Figure 5). Adjust the high voltage until you see a spurious/random pulse above a flat background. The exact setting is not critical;
however, the voltage should be monitored using a digital multimeter set to DC voltage. The probe connectors are at the top of the detector tube (note that the measured voltage is divided by 100).

4) Connect the PMT output to one of the channels on the digital oscilloscope. To see a signal you need to turn on the PULSER switch on top of the detector. Be certain to terminate the input to the oscilloscope with the BNC 50Ω terminator, or your signal will be distorted. Record and label the oscilloscope trace.

5) Connect the BNC cable between PMT Output on the detector and PMT Input on the electronics box. Using a digital multimeter, adjust the discriminator setting on the electronics box so that it is in the range 180 mV - 220 mV. The green LED on the box front panel should now be flashing.

6) Examine the amplifier output using a digital oscilloscope - the Amplifier Output is located on the front panel of the electronics box. Be certain to terminate the input to the oscilloscope with the BNC 50Ω terminator. Record and label the oscilloscope trace.

7) Similarly, examine the output from the discriminator using a digital oscilloscope; the Discriminator Output connector is located on the front panel of the electronics box. Again, a BNC 50Ω terminator must be used. It is useful to simultaneously display the amplifier and discriminator outputs on the one screen using a two channel digital oscilloscope.

**Instrumentation Exercises**

1) Measure the gain of the 2-stage amplifier (using a sinusoidal input), by applying a 100 kHz 100mV peak-to-peak sine wave to the PMT input of the electronics box. Measure the amplifier output and take the ratio $V_{\text{out}} / V_{\text{in}}$. Due to attenuation resistors inside the electronics box, inserted between the amplifier output and the front panel connector, you will need to multiply this ratio by the factor 1050/50 = 21 to determine the real amplifier gain.
   a) Adjust the frequency and plot the transfer function, i.e., $V_{\text{out}} / V_{\text{in}}$ versus frequency. How good is the frequency response of the amplifier? Note: for this measurement make sure that the probe multiplier is 1x on the oscilloscope.
   b) Estimate the maximum decay rate you could observe with the instrument.

2) Measure the saturation output voltage of the amplifier, by increasing the magnitude of the input sine wave and monitoring the amplifier output.
   a) Does a saturated amplifier output change the timing of the FPGA? What are the implications for the —magnitude of the signals from the scintillator?

3) Examine the behavior of the discriminator by applying a sine wave to the PMT input and adjusting the discriminator threshold. Monitor the discriminator output and describe its shape.

4) Measure the timing properties of the FPGA. For this part you will use the pulser on top of the detector and the “rate meter” function that appears after the *muon* program on the computer.
   a) Measure the time between successive rising edges on a digital oscilloscope. You can compare this number with the number shown on the “rate meter” software display.
   b) Measure the linearity of the FPGA, by altering the time between rising edges, and plotting your results against the FPGA results. Use time intervals between 1μs and
20\textmu s in steps of 2\textmu s.

c) Determine the timeout interval of the FPGA by gradually increasing the time between successive rising edges of a double-pulse and determine when the FPGA no longer records results. What does this imply about the maximum time between signal pulses?

d) Decrease the time interval between successive pulses and try to determine the FPGA internal timing bin width. What does this imply about the binning of the data? What does this imply about the minimum decay time you can observe?

5) Adjust (or misadjust) the discriminator threshold, by increasing the discriminator output rate as measured by the digital oscilloscope. Observe the raw muon count rate and the spectrum of —decayl times. N.B. This exercise needs some patience since the counting rate is slow.

6) Investigate the effect of the HV setting, by adjusting the HV and observing the amplifier output. We know that —goodl signals need to be at about 200 mV before being input to the discriminator, so set the discriminator before hand. With fixed threshold, alter the HV and monitor the raw muon count rate and decay spectrum.

Measuring the Muon Lifetime

1) Open the folder/directory muon_data and launch the program muon.exe. Configure the port on your PC - make sure you select Com4. Choose your histogram options and click on the Save/Exit button. To start data acquisition click on Start. You should see the rate meter at the lower left-hand side of your computer screen immediately start to display the raw trigger rate for events that trigger the readout electronics. The mean rate should be about 6 Hz.

2) Set up the instrument for a muon lifetime measurement. Before you start data acquisition it is critical to ensure you have the correct voltage settings for the PMT use the Calibration above for the HV setting and the discriminator setting. Press Start and observe the decay time spectrum.

Note that in order to acquire a statistically significant number of events (>1500) it will be necessary to acquire data over a period of two or more days. Do not connect/disconnect USB items or switch attached equipment on and off while you are collecting data as this may cause electrical noise which can give rise to false muon events.

Important: disconnect computer from internet while collecting data.

3) Use the lifetime fitter function in the Muon control panel to fit the decay time histogram. A Screen capture button allows you to produce a plot of the display. Select the button and then open the Paint utility (in Windows) and execute the Paste command under the Edit pull-down menu.

4) You should also investigate your own fit to the data due to limitations of the Muon software. The file manual-data-analysis on the course website outlines the data fitting process in ORIGIN. It is also possible to analyze the data using other software, such as Excel or Matlab, using a similar protocol to that outlined.

a) Compare your measured value of the muon lifetime with that reported in the literature, and that reported by the muon lifetime fitter.
Question. Why in the manual data analysis process is it suggested to discard the first bin in the histogram before fitting?

5) From your measurement of the muon lifetime calculate the value of the Fermi coupling constant \( G_F \). Compare your value with a literature value.

6) Using the approach outlined in Sec. 4.2, measure the charge ratio of positive to negative muons at sea level. Compare to value in the literature.

Exercise:
Once the muon lifetime is determined, compare the theoretical binomial distribution with an experimental distribution derived from the random lifetime data of individual muon decays. For example, let \( p \) denote the (success) probability of decay within one lifetime, \( p = 0.63 \). The probability of failure \( q = 1 - p \). Take a data sample of 2000 —“good” decay events. For each group of 50 events, count how many have a decay time less than one lifetime. (On average this is 31.5.) Histogram the number of —‘successes’. This gives you 40 —‘experiments’. The plot of data from these 40 —‘experiments’ should have a mean of 50×0.63, with a variance \( \sigma^2 = Npq = 50 \times 0.63 \times 0.37 = 11.7 \).

Are the experimental results consistent with theory?

**Discussion and questions to include in lab report**

1. The muons whose decays we observe are born outside the detector and therefore spend some (unknown) portion of their lifetime outside the detector. So, we never measure the actual lifetime of any muon. Yet, we claim we are measuring the lifetime of muons. How can this be?

2. At sea level, what is the total flux of muons and what is their average energy in GeV? Given that the altitude of Serin is ~ 19m above sea level, estimate the number of muons that enter your detector per unit time.

   This should allow you to estimate the expected number of decays per unit time in your experiment. Compare this number to the rate you measured in the experiment. Discuss your result.

3. Charged particles such as muons lose energy when traversing matter.

   According to the Bethe-Bloch formula this energy loss is roughly constant for high energy particles at about 2 MeV g\(^{-1}\) cm\(^2\). Estimate the energy loss of muons in the atmosphere before reaching your detector? Note that 1 atm \( \sim 10^5 \) N/m\(^2\).

4. Do you expect to measure the same lifetime for positive and negative muons? What are muonic atoms and why are negative muons much likelier to be captured by the nucleus than electrons? The following article will help to understand this problem: T. Suzuki, Total nuclear capture rates for negative muons, Physical Review C35 (1987) 2212.

5. From the measured muon lifetime calculate the ratio of positive to negatively charged muons that arrive at your detector.

6. From your measurement of the muon lifetime and a value of the muon mass from some trusted source, calculate the value of Fermi coupling constant \( G_F \). Compare your value with accepted values.

7. A muon decays according to

\[
\mu^+ \rightarrow e^+ + \nu_e + \nu_e
\]
What is the maximum momentum (in units of MeV/c) that the electron can have, assuming the muon was initially at rest?
The muon mass is about 105 MeV; mass of the electron is about 0.5 MeV. Assume neutrinos of all three types (electron, muon and tau) are massless, although this is currently in question.) What is the minimum momentum the electron can have?
References
References

1. Serway, Moses and Moyer: Modern Physics, p 13  (lifetime of moving muon); pp444-456 (elementary particles, leptons)

2. D.H. Perkins, Introduction to High Energy Physics,


