The Optical Zeeman Effect

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(Dated: 12 October 2009)

The Optical Zeeman Effect in mercury and neon is observed using a SPEX 1000M spectrometer. The Landé g-factors for the transitions are calculated from the data and compared with predicted values. The observed values agree with theory, but the errors in the measurements are large due to poor resolution.

Introduction

The Zeeman Effect is the splitting of spectral lines due to an applied magnetic field. It was looked for by Faraday, who failed to observe the effect due to resolution. In 1896, Pieter Zeeman observed the splitting of the sodium yellow D lines.

Background

An atomic spectral line originates from the de-excitation of electrons through radiation of a photon. A shift in this photon's energy is due to a shift in the excited state, the ground state, or both from an applied magnetic field. An electron with magnetic moment $\boldsymbol{\mu}$ in an applied field **B** experiences an energy shift given by:

$$E = -\mu \cdot \mathbf{B}$$

The Normal Zeeman Effect

The Normal Zeeman Effect refers to the splitting of transitions in which the spectral line has split into a triplet under the influence of an applied external field. This occurs when the effective Landé g-factor is 1. An electron in a singlet state has **S**, the spin angular momentum, equal to zero so **J**, the total angular momentum, is **L**. Upon consideration of this fact along with selection rules and quantization:

$$\Delta E = -m_J \, \mu_B B$$

The levels split evenly, so the spectral line is split into 3 evenly spaced ones. We refer to the case when **S**, the spin angular momentum, is non-zero as the Anomalous Zeeman Effect.

The Anomalous Zeeman Effect

The Anomalous Zeeman Effect refers to the case where **S**, the spin angular momentum, is non-zero in either the initial or final state. (Or equivalently when the effective Landé g-factor is not 1.) We find that this leads to a different energy splitting, for a single state's energy, given by:

$$\Delta E = -m_J g \mu_B B$$

where g is the Landé g-factor:

$$g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$$

which leads to the change in observed photon energy given by:

$$\Delta E_\gamma = g_{eff} \mu_B B$$

where g_{eff} is the effective Landé g-factor mentioned previously and is given by:

$$g_{eff} = g_{final} m_{J_{final}} - g_{initial} m_{J_{initial}}$$

where initial and final mean the initial and final states of the transition. These transitions are polarized in the following manner:

$$\Delta m_J = \pm 1, \qquad \sigma\text{-polarized}$$
$$\Delta m_J = 0, \qquad \pi\text{-polarized}$$
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σ-polarized light is circularly polarized when observed parallel to the applied magnetic field and linearly polarized perpendicular to the field when viewed at right angles to the field[1]. π polarized light is plane polarized with the direction of polarization parallel to the applied magnetic field[1]. The change in the photon energy will result in a splitting of the spectral lines of magnitude:

$$\Delta\lambda = g_{eff}\left(\frac{\mu_B \lambda^2}{hc}\right) B$$

where $\Delta\lambda$ is measured from the original unsplit line.

Apparatus and Procedure

In this lab a SPEX 1000M spectrometer was used to observe the spectral lines and splitting. A basic schematic is shown below in Fig. 1.

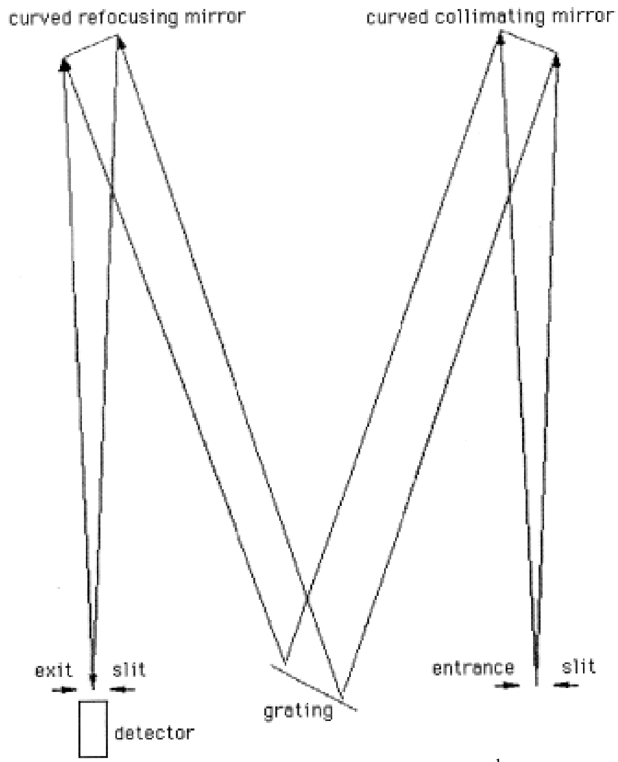

Figure 1-Schematic of spectrometer[1]

There were actually two detectors on the apparatus, one on the front, which had lower resolution but could quickly perform measurements over a large range of wavelengths, and a side detector, which had finer resolution but could only observe a small range of wavelengths at a time. The side detector was used exclusively in this experiment because the splitting was small.

The procedure simply consisted of placing a gas tube of either mercury or neon between the poles of an electromagnet. The zero-field line would be located initially and then the electromagnet would be turned on and the split line would be examined. The magnitude of the magnetic field was measured with a calibrated hall probe.

The following four lines were examined in detail:

Hg 5461 Å	$^3S_1$ to $^3P_2$
Hg 4358 Å	$^3S_1$ to $^3P_1$
Hg 4047 Å	$^3S_1$ to $^3P_0$
Ne 5853 Å	

where the transition is not listed in neon because the effect is much more complicated. The σ and π polarizations were examined for each line. The data was recorded on a computer and the necessary analysis was performed there.

Analysis

The effective g-factor, g_{eff}, defined above can be calculated directly by the above equation once one has measured the splitting resulting from a magnetic field of known strength. Alternatively, one can calculate g_{eff} by measuring the splitting for various magnetic fields. The value for g_{eff} can then be calculated from the slope of the best fit line. This method was used on the σ-line of the 4047 Å line of mercury and both polarizations of the 5853 Å line of neon. The advantage of finding g_{eff} from multiple measurements is that the percent error should be reduced. The resulting graph for the mercury 4047 Å σ-line is shown below in Fig. 2. The other graphs are not shown in the text but look similar to Fig. 2.

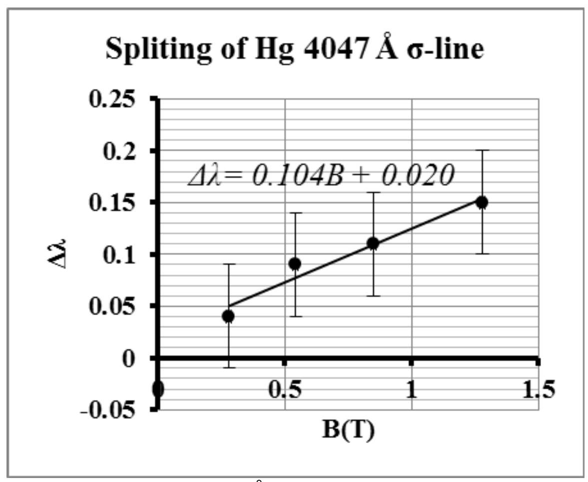

Figure 2-Hg 4047 Å σ-line g-factor best fit graph

Since the slope of this line is:

$$g_{eff}\left(\frac{\mu_B\lambda^2}{hc}\right)$$

One must simply divide the slope by the constant to obtain g_{eff}. Table 2 shows the results of our analysis.

Line	g_{eff}
Hg 4047 Å (σ)	1.4 ± 0.9 ($g_{theoretical} = 2$)
Ne 5853 Å (σ)	0.9 ± 0.5

Ne 5853 Å (π)	0.8 ± 0.5

Table 1

The theoretical value of g_{eff} is shown for the mercury transition.

The errors in these values were calculated by the standard formula for the error in the slope of a best fit line. The individual errors in our measurements of B and $\Delta\lambda$ were $\pm 0.01\ T$ and ± 0.05 Å respectively. The relatively large error in the wavelength is the resolution of the instrument (full width at half max of the peaks). It is very large when one considers that the splitting of the spectral lines themselves is on the same order of magnitude, but on can see from Fig. 3 below that our graphs are consistent with what one would expect with a detector of this resolution. Fig. 3 depicts a simulation using Lorenzian line shape for the lines of the π-polarization for the $^3S_1$ to $^3P_1$ transition followed by our data of the same transition.

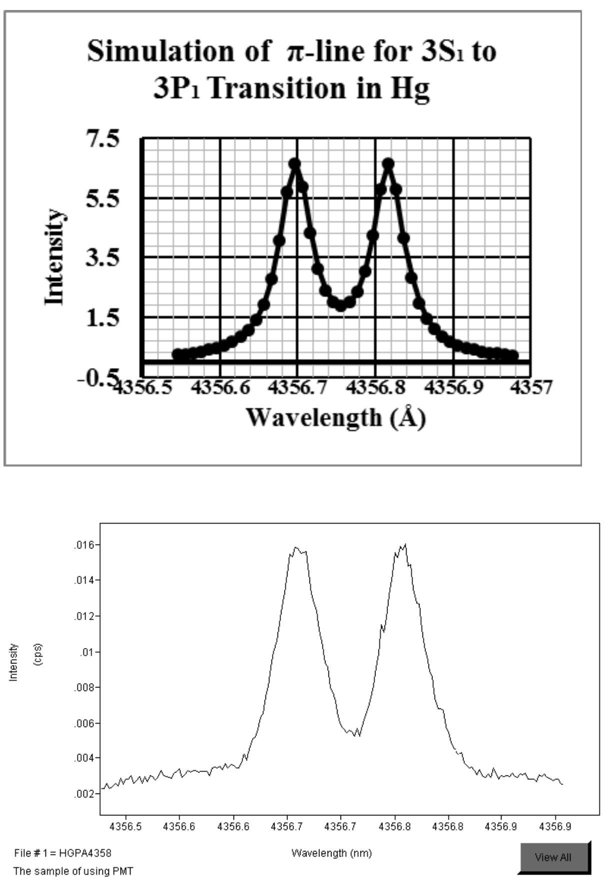

Figure 3- Simulated and observed splitting. The scale is the same on the two graphs

Table 2 shows observed and predicted splittings for the mercury $^3S_1$ to $^3P_1$ transition, with the stated distances being the change in wavelength from the unsplit line.

	Observed Splitting (Å)	Predicted Splitting (Å)	Difference (Å)
π spectrum	0.04 ± 0.05	0.07	0.03±0.05
σ spectrum ($g_{eff} = \pm 3/2$)	0.16±0.05	0.20	0.04±0.05
σ spectrum ($g_{eff} = \pm 2$)	-	0.27	-

Table 2 – Observed and Predicted splitting for Hg 3S1 to 3P1 transition

Using this data, an effective Landé g-factor was calculated for the π-splitting using the equation:

$$g_{eff} = \frac{\Delta\lambda \cdot hc}{\lambda^2 \mu_B B}$$

This was found to be $g_{eff,\pi} = 0.4 \pm 0.2$. This result compares favorably with the predicted value of 0.5. The large percent error in this result is mostly due to the limited resolution of the spectrometer as discussed above, and this value could be calculated more precisely if a spectrometer with finer resolution were used.

The experimental σ-line spectrum for the $^3S_1$ to $^3P_1$ transition is shown here in Fig. 4. A magnetic field of 1.43 ± 0.01 T was applied during this measurement. The theoretically expected spectrum is also shown in Fig. 5, for two different spectrometer resolutions.

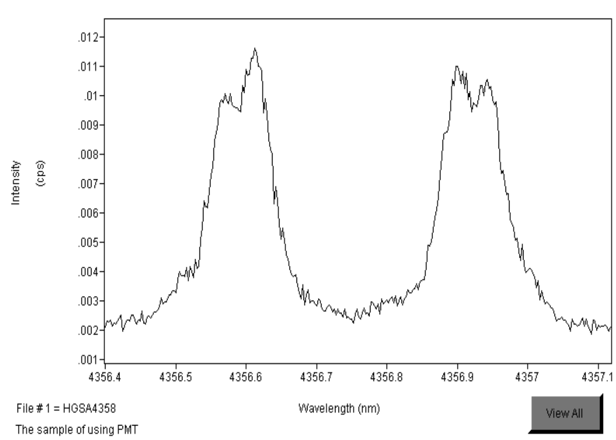

Figure 4 – Hg 4358 Å σ spectrum

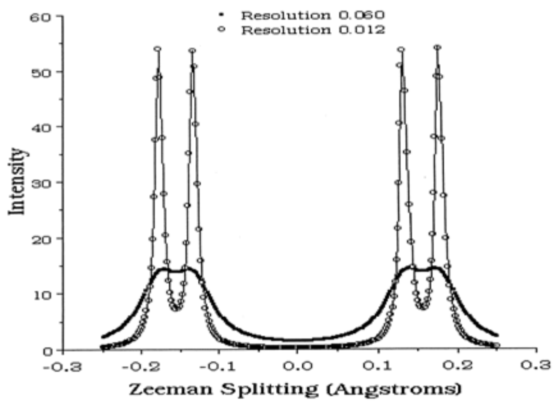

Figure 5 – Hg 4358 Å σ simulation[1]

Given the limited resolution of the spectrometer used, the qualitative features of the experimental spectrum compare favorably with the theoretically predicted spectrum. The four separate peaks of the split σ-lines are just beginning to resolve, and would likely be visible if a spectrometer of greater resolution were used. It is important to note that the location of the widened peaks agrees very well with the simulation.

An effective Landé g-factor was also calculated for this σ-splitting, using the distance between the center of the two fully resolved peaks as the best measured value for the splitting. This calculated value was $g_{eff,\sigma} = 1.3 \pm 0.2$. This value compares favorably with the theoretically expected values of $g_{eff,\sigma} = \frac{3}{2}, 2$ for the inner and outer splittings, respectively.

The π and σ spectra were also measured for the Hg 5461 Å line, corresponding to the $^3S_1$ to $^3P_2$ transition. These spectra are shown in Fig. 6 and Fig. 7, respectively.

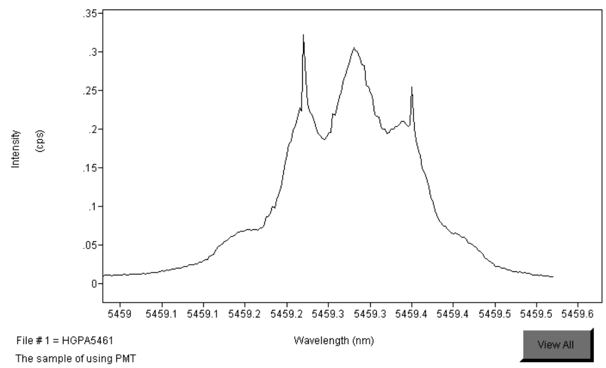

Figure 6 – Hg 5461 Å π spectrum, magnetic field of 1.40 ± 0.01 T

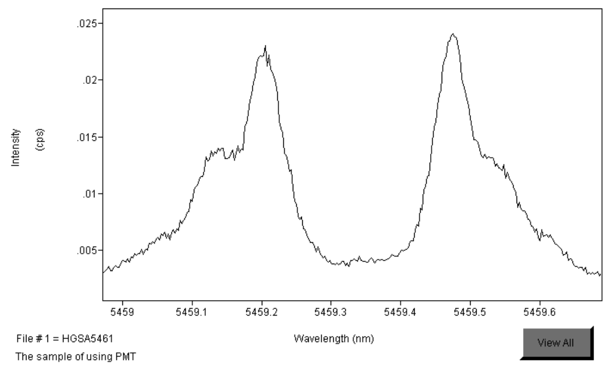

Figure 7 – Hg 5461 Å σ spectrum, magnetic field of 1.48 ± 0.01 T

The theoretically predicted effective Landé g-factors and corresponding wavelength shifts for Hg 5461 Å in a 1.5 T magnetic field are shown here in Table 3. Forbidden transitions are excluded.

$m_{j'}$	m_j	Δm_j	polarization	g_{eff}	$\Delta\lambda$ (Å)
-1	-2	-1	σ	-5/2	-0.52
-1	-1	0	π	-1/2	-0.10
0	-1	-1	σ	-2	-0.42
-1	0	1	σ	3/2	0.31
1	0	-1	σ	-3/2	-0.31
0	1	1	σ	2	0.42
1	1	0	π	1/2	0.10
1	2	1	σ	5/2	0.52

Table 3– Expected transitions, effective g factors, and wavelength shifts of Hg 5461 Å in 1.5 T field

Qualitatively, the experimental π spectrum for Hg 5461 agrees very well with the predicted results. Two well resolved peaks were observed, each with a wavelength difference of approximately 0.10 Å from the unsplit center line. However, this spectrum has an asymmetric shape in that one peak has much higher intensity than the other. This is likely due to the spectrometer measuring the lower wavelength peak at or near its maximum intensity, while measuring the higher wavelength peak at two points surrounding but not as near to the maximum intensity as the lower wavelength peak.

Qualitatively the experimental σ spectrum of the Hg 5461 Å line also agrees well with the theoretically predicted spectrum, if the limited resolution of the spectrometer is taken into account. While only two of the six predicted peaks are observed to be fully resolved, these peaks are wide when compared to the peaks observed for the other spectra, from which it may be inferred that these observed peaks are not singular peaks, but unresolved multiple peaks. The observed peaks are also seen to be beginning to resolve into multiple peaks, particularly on the lower wavelength peak, which contains two obvious local maxima.

Effective Landé g-factors were also calculated for the experimental spectra of the π and σ spectra for the Hg 5461 Å line. The calculated effective Landé g-factor for the π spectrum was 0.4 ± 0.1, which compares favorably with the theoretical effective Landé g-factor of 0.5. The calculated effective Landé g-factor for the σ spectrum was 1.8 ± 0.2, which falls between the expected Landé g-factors of 3/2 and 2, and thus corresponds well with theory, when the limited resolution of the spectrometer is taken into account.

Conclusion

All of the experimentally determined g-factors determined in this lab agree very well with experiment within the error associated with the resolution of the spectrometer. With the spectrometer that was used in this lab, it is not possible to obtain any better agreement. The resolution was consistent throughout the measurements and did not change by varying the parameters that the spectrometer was using to collect data. The optimal values of these parameters, as far as could be determined, were used in this lab. The qualitative agreements between the predicted and observed splitting amounts were excellent especially when one considers the resolution of the spectrometer. That being said, many of the large errors in splittings cannot rule out the normal Zeeman effect and if one wished to rule it out completely by experiment, one would need a spectrometer with better resolution.

References

1. Zeeman Effect Lab Manual
2. A. Melissinos and J. Napolitano. Experiments in Modern Physics, 2nd ed. New York: Academic Press, 2003.