

THE FRANCK-HERTZ EXPERIMENT

OBJECT: To measure the excitation potential of mercury using the Frank-Hertz method.

EQUIPMENT LIST: Electrometer (Keithley model 60), digital multimeter (Keithley model 160), 0-300 V DC power Supply (Klinger Scientific), mercury vapor triode and oven (Neva-Klinger Scientific), rheostat, multimeter, digital storage oscilloscope (Hewlett-Packard model 54600A)

THEORY: 1. Excitation by electron impact of quantized, bound atomic states. In 1913 Niels Bohr proposed the Bohr model of the atom which assumes that atoms can exist only in certain bound energy states. This idea was given a powerful boost in 1914 when James Franck and Gustav Hertz performed an experiment that demonstrated the existence of quantized energy levels in mercury. The experiment involved sending a beam of electrons through mercury vapor and observing the loss of kinetic energy when an electron strikes a mercury atom and excites it from its lowest energy state to a higher one.

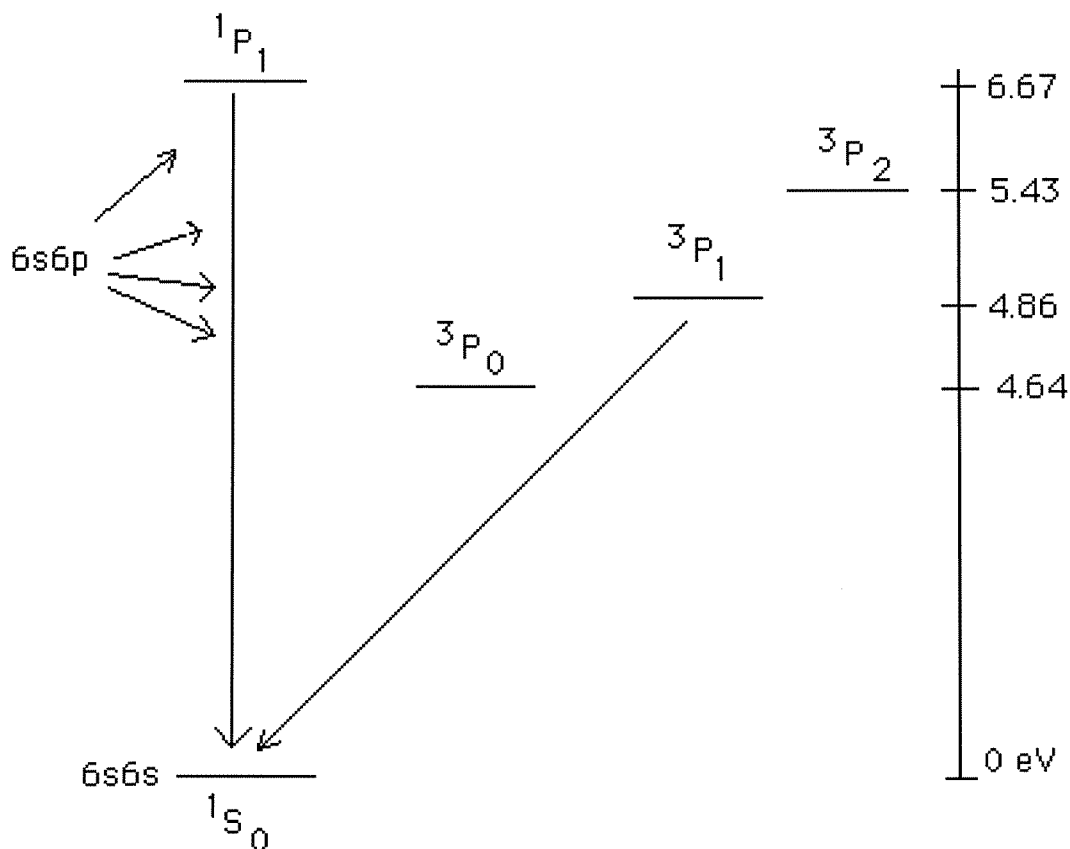


Figure 1: Energy Levels of Atomic Mercury

Mercury vapor atoms will normally be in their lowest or ground state, with the two valence electrons occupying a state designated by $(6s)^2$ (two electrons in $n = 6$, $l = 0$ single particle states). The two electrons do not move independently so that three quantities -- the total spin angular momentum S , the total orbital angular momentum L ,

and the total angular momentum J -- are constants of the motion designated by quantum numbers S , L , and J , respectively. Thus the electron states are labeled with the spectroscopic notation $2S+1L_J$. The value of L is denoted by S for $L = 0$, P for $L = 1$, etc. The ground state in mercury is then $1S_0$.

As shown in Fig. 1, the next levels above the ground state are a "triplet" of levels -- $3P_0$, $3P_1$, and $3P_2$ -- corresponding to single electron states ($6s6p$) where the electron spins are parallel. There is a higher singlet $1P_1$ state with spins antiparallel. In a collision with an energetic electron the atom could be raised into any of these excited states.

However, in the Franck-Hertz experiment we only observe excitation into the $3P_1$ state for the following reason: Once the atom is in an excited state it can return to the ground state by emission of a photon. This is normally a very rapid process typically taking 10^{-8} s. However, photon de-excitation transitions must satisfy conservation of total angular momentum. The emitted photon has an angular momentum of 1 which it carries away from the atom. Thus the total angular momentum of the atom must change by 1, $\Delta J = \pm 1$ (remember angular momentum is a vector). Thus de-excitation can occur from the $3P_1$ and $1P_1$ states, but not from the $3P_0$, and $3P_2$ states, which are metastable -- de-excitation from a metastable state can only occur by slower processes which typically take on the order of 10^{-3} s. In the Franck-Hertz experiment the electron beam may excite a mercury atom into the $3P_0$ or $3P_2$ state, but then it is stuck there (for a millisecond) and unable to absorb more energy. On the other hand, if the $3P_1$ state is excited, it quickly de-excites (in 0.01 microseconds) and the atom is again available to absorb energy from the electron beam. The $1P_1$ state is not observed since the $3P_1$ state is so effective at taking energy from the electron beam once the electrons reach 4.86 eV that they are not able to acquire the 6.67 eV needed to excite the $1P_1$ state.

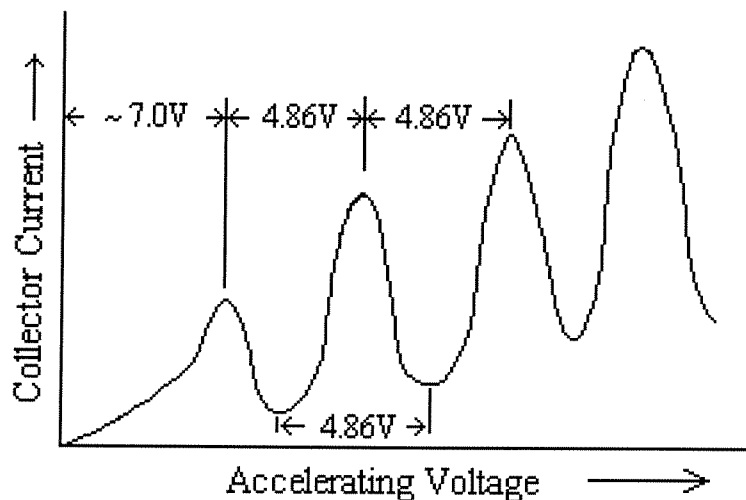


Figure 2

In this experiment electrons emitted by a hot cathode are accelerated through the mercury vapor to a collector electrode. When the accelerating voltage is increased to 4.86 V, the collector current will drop due to the onset of energy loss of the electrons caused by

collisions with Hg atoms that raise the atoms from their 1P_0 ground state to their 3P_1 excited state. If the voltage is further increased the current will again increase until $2 \times 4.86 = 9.72$ V where it will drop again, as shown in the sketch on the previous page. This is due to electrons which having lost most of their kinetic energy in the first collision again being accelerated to 4.86 eV kinetic energy so that in a second they can again excite the 3P_1 state. This process will be repeated for 3×4.86 V, 4×4.86 V, etc. As we will discuss below there are several effects that will shift this pattern of repeated valleys (or peaks, depending how you look at the graph) to higher voltages. However, the spacing between the peaks/valleys will remain equal to 4.86 eV. Excitation of states of lower and higher energy than 4.86 eV, if seen at all, will appear as "shoulders" on the dominant current drop.

2. Mean free path of electrons. The phenomena involved in this experiment are influenced strongly by how far, on the average, an electron goes before colliding with a vapor atom, and producing excitation. The average distance which an electron travels between collisions, **of any type**, is called the **electron mean free path** $\bar{\ell}$. This can be estimated from :

$$\bar{\ell} = \frac{4/\sqrt{2}}{n(\pi R_o^2)} \quad (1)$$

R_o is the radius of a Hg molecule and n is the number density of the gas, i.e. the number of molecules per unit volume. (See Halliday and Resnick for a derivation.) Note from Eq. (1) that $\bar{\ell}(\pi R_o^2)n \approx 1$. This means that $\bar{\ell}$ is the average distance an electron will travel before coming within R_o of a mercury atom. In mercury the atomic radius is about 0.15 nm = 1.5×10^{-8} cm.

In this experiment the Franck-Hertz tube, which contains a drop of mercury, is placed in a heated oven. The mercury drops evaporates until the vapor is saturated, i.e. the rate of evaporation equals the rate of condensation. By changing the temperature T of the oven, you will be able to change the vapor pressure and hence the density. To estimate the mean free path, we will need to calculate $n(T)$. Mercury is monatomic. From the definition of gram-atomic weight A , the number of molecules per cubic centimeter is

$$n = (\text{mass}/\text{cm}^3)/(\text{mass}/\text{atom}) = \rho \div (A/N_A) = \rho N_A/A \quad (2)$$

where ρ is the mass density of the vapor and $N_A =$ Avogadro's number.

The Handbook of Chemistry and Physics or International Critical Tables give the pressure of saturated mercury vapor at various temperatures. If it is assumed that the vapor behaves approximately as an ideal gas at these low temperatures, then the density can be calculated by using the general gas law

$$PV = (m/A)RT, \quad (3)$$

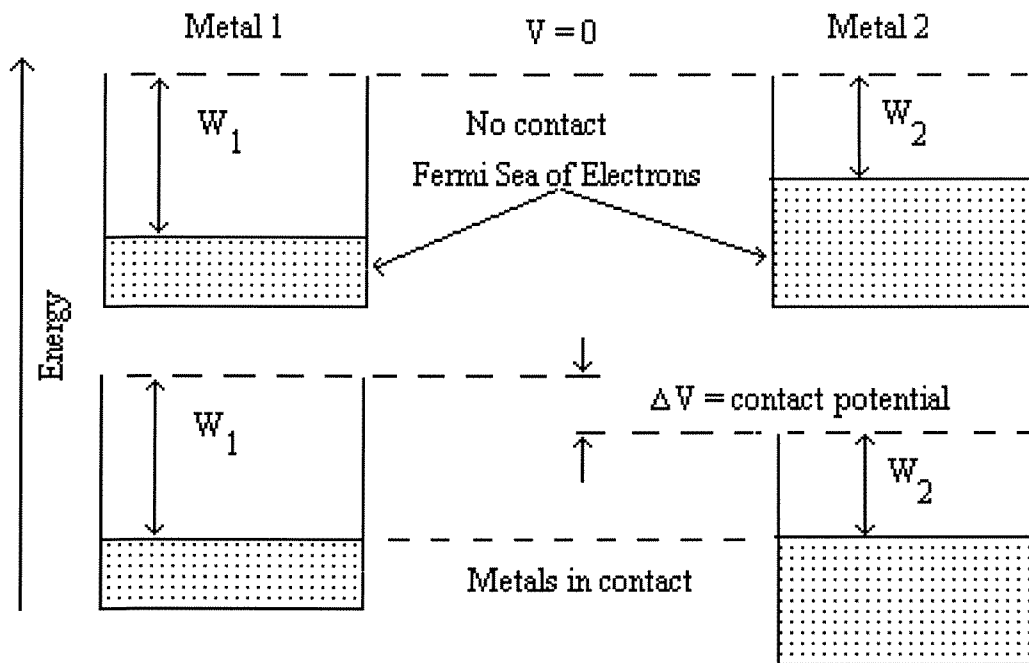
where m is the mass of gas in the volume V and m/A is the number of moles. Then using

$\rho = m/V$ in Eqs. (2) and (3) we find

$$n = \frac{N_A}{R} \frac{P}{T}. \quad (4)$$

In using this equation, be sure that you use a value for R expressed in the same units you use for P and that T is in Kelvins. Then you can estimate the mean free path at any oven temperature by using Eq. (4) in Eq. (1) along with the tabulated saturated vapor pressure $P(T)$. This table is given in the binder of readings available in the lab.

When taking data to determine the ionization potential, it is necessary to have the mean path for collisional excitation of the same magnitude or longer than the distance d from the cathode to the grid, but smaller than the distance between cathode and anode. On the other hand, when taking data to determine the excitation potential, it is necessary to have this mean path short compared with this same distance, in order to see several peaks and valleys. For your lab vary the oven temperature to optimize your data and for your report you will prepare a theoretical plot of $\bar{l}(T)$.



3. Contact potential. We saw in the discussion of the photoelectric effect that the work function W is equal to the photon energy required to extract an electron from a substance with zero final kinetic energy. The top portion of the diagram above illustrates the concept of the work function for two metals: In a metal, the valence electrons are not bound to the metallic ions but can wander freely throughout the metal and can be thought of as a (Fermi) sea of electrons filling a box to a certain height. The work function is the minimum energy to remove an electron from the top of this sea to a point far from the metal. If the two metals shown in the diagram are placed in contact electrons flow from one to the other until energies of the top of the two Fermi seas (called the Fermi levels)

are equalized. It only takes a few electrons to do this. A potential difference ΔV then develops between the two metals which is called the contact potential, as shown in the diagram. From the diagram we see $\Delta V = W_1 - W_2$. For the Franck-Hertz tube the manufacturer indicates that the contact potential is about 2 V, which you will be able to verify with your measurements. Note that to have a contact potential, the two metals do not have to be in intimate physical contact; it is sufficient for there to be a way for electrons to travel from one metal to the other. In the Franck-Hertz tube the electrons travel through the vacuum from the cathode to the anode; the two metals do not “touch”.

The electrons that travel through the Franck-Hertz tube are released by **thermionic emission** from the very hot cathode as follows: The surface of the Fermi sea that we have shown in the diagram is appropriate for a temperature of absolute zero. At higher temperatures thermal agitation raises some of the electrons above the Fermi level. At room temperature the average thermal energy is $kT \sim 0.025$ eV, which is not enough for electrons to overcome the work function and escape. But when the cathode is raised to a high temperature (~ 1500 K) a few electrons have enough energy to escape, i.e. to be thermionically emitted. Ideally for the Franck-Hertz experiment we would like these electrons to have zero kinetic energy so that this initial kinetic energy will not interfere with our measurement of the change in kinetic energy due to inelastic collisions. However, experimental measurements show that for cathode temperatures in the 1500 K to 2500 K range thermionic electrons have a distribution of kinetic energies peaking at 0.2 to 0.3 eV. This will affect the location of the first peak in the current, but not the spacing of subsequent peaks.

EQUIPMENT: The essential components are a mercury thyratron tube, a variable power supply for the cathode heater, anode and collector voltage supplies with necessary meters, a sensitive dc amplifier for observing the small collector currents, and an oven with thermometer and controls to heat the thyratron to an appropriate temperature to produce the desired vapor pressure.

1. The oven. The oven consists of a small steel cabinet with a heating element to uniformly heat the tube (and all connections leading to it). A 300 watt heating element is mounted on the bottom of the housing. An adjustable thermostat in the oven regulates the temperature of the oven. Since the thermostat functions imperfectly (1-3 degrees drift), it is preferable to bypass it by setting it to maximum temperature and instead limiting the power delivered to the oven with a Variac (a variable auto transformer that steps down the 110 V AC line voltage). This approach gives adequate stability. A hole in the top of the cabinet is provided for a thermometer.

2. Thyratron tube. A thyratron is a vacuum tube in which a drop of mercury was added before sealing. Such tubes, therefore, contain saturated mercury vapor (vapor in equilibrium with the liquid) at a pressure corresponding to the temperature of the bulb. The Franck-Hertz tube, a triode, has three plane-parallel electrodes which provide a uniform electric field to accelerate the electrons. The electrodes are the cathode (K), anode (A) and collector (M). The cathode is indirectly heated with a heater electrode (H and K) using a nominal voltage of 6.3 volts AC. (Note that the rheostat used to vary the

cathode heater current should be connected to H, not to K which is grounded.) The anode is a perforated screen 8 mm beyond the cathode that is held at a positive potential V_a relative to the cathode in order to accelerate the electrons thermionically emitted by the heated cathode. The screen allows most of the electrons to pass through the anode. The collector lies a small distance (~ 2 mm) beyond the anode and is negatively charged relative to the anode with a voltage V_{ret} , which acts to retard (slow) the electrons that pass through the anode. These electrons (charge e) can make it to the collector, if as they pass through the anode their kinetic energy is greater than eV_{ret} . Otherwise, they slow, stop, and reverse direction to return to the anode.

3. Operation of the Franck-Hertz tube to observe inelastic scattering through changes in the collector current. There are two lengths of interest -- the cathode anode separation (8 mm) and the mean free path $\bar{\ell}(T)$ -- and two voltages -- the accelerating voltage V_a between the anode and cathode and the retarding voltage V_{ret} between the anode and collector. It is important to understand how they affect the experiment.

The value of $\bar{\ell}$ and hence the temperature of the oven is not critical as long as $\bar{\ell} \ll 8$ mm since you want to be sure the electrons make collisions with mercury atoms in the space between the cathode and anode. In order for this condition to hold, you will take data in the temperature range 180° - 195° C. [On the other hand, you do not want collisions between the anode and collector, so the tube is built with these electrodes closely spaced, ~ 2 mm.]

As shown in the sketch on page 2, the collector current will show a series of equally spaced peaks as the accelerating voltage V_a is increased. For a given V_a the number of times n_{col} that an electron can make an inelastic collision with a mercury atom and excite it to its 3P_1 state (and losing 4.86 eV of kinetic energy in the process) while in the space between the cathode and anode is determined by the ratio

$$n_{col} \leq \frac{V_a}{4.86}, \quad (5)$$

where n_{col} is the largest integer smaller than the ratio. As shown in the sketch, this relation has to be slightly corrected for two reasons. First, before an electron can make its first inelastic collision, it must overcome the cathode to anode contact potential of about 2 eV. Second, there is a small shift (toward lower V_a) due to the small (~ 0.2 to 0.3 eV) initial kinetic energy of thermionically emitted electrons. Note that the contact potential and the initial kinetic energy only affect the value of V_a needed for the first inelastic collision; successive peaks will be separated by 4.86 eV.

Also note that although the onset in inelastic scattering is governed by Eq. (5), the drop in current is not sharp. This is because the excitation cross section is small at the threshold of excitation (kinetic energy = 4.86 eV) and the drop in current due to inelastic scattering doesn't reach a maximum until the kinetic energy is larger, say 5.1 eV. Note however, that the spacing of successive minima in the collector current should still be 4.86 volts: If

peak excitation of the 4.86 eV state occurs at 5.10 eV, then an electron inelastically scattered with 5.10 eV will still have 0.24 eV of kinetic energy after collision. After accelerating through another 4.86 V, the electron will again have 5.10 eV of kinetic energy.

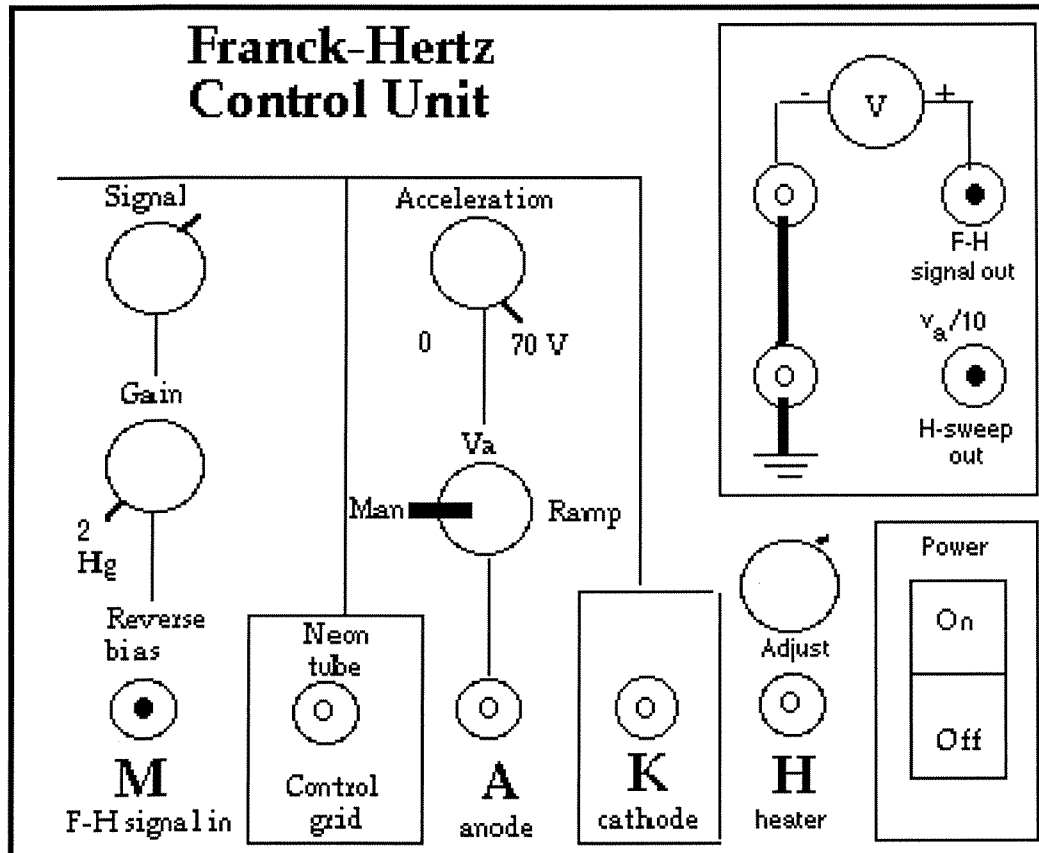
As one scans the accelerating voltage up to the maximum V_a allowed by the power supply, fewer peaks may be observed than predicted by Eq. (5), if (due to low oven temperature) the mean free path for inelastic scattering exceeds the distance required for the electron to attain 4.9 eV. The number of peaks observed will vary with oven temperature; experimentally you will want to vary the temperature to optimize the pattern of peaks.

The final relation we need to discuss is the role that the retarding voltage V_{ret} plays in the experiment, which is to generate the valleys! The current in the tube is proportional to the product of the electron density and the mean electron speed. If the tube had only two electrodes, you would not observe deep drops in current; these are produced by V_{ret} . When V_a is such that electrons undergo inelastic scattering just before passing through the anode, then they do not have enough kinetic energy to overcome the retarding effect of $V_{ret} \sim 1.5$ V. The retarding voltage will then turn them around and prevent them from reaching the collector, producing the sharp drop in current (as a function of V_a). We thus see that the locations of the peaks and valleys are independent of the value of V_{ret} , although the depth and shape of the valleys will depend on V_{ret} .

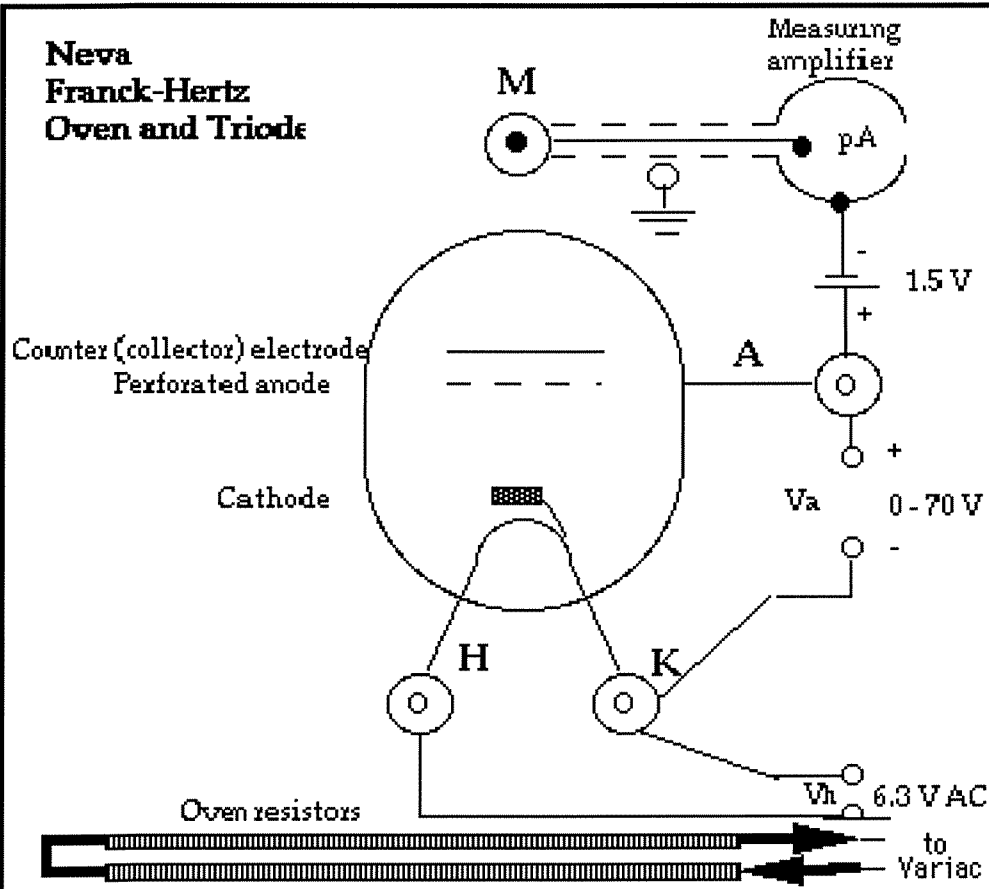
PROCEDURE:

1. The Franck-Hertz control unit and triode are shown schematically on the next page. Wire the apparatus by connecting terminals **M** to **M**, **A** to **A**, etc. with a rheostat and AC ammeter in series with the **H** (heater) input to allow heater current variation. [Note: When connecting banana plug outputs to BNC cables, be sure to observe proper grounds on the connectors; the little tab on the plastic banana plug labeled “gnd” indicates that that side of the banana plug is connected to the coaxial cable shielding, therefore to the ground of the coax BNC connector.]
2. Connect the F-H signal out terminal to the channel 1 input of the digital scope and the $V_a/10$ output to channel 2. If the scope is set to X-Y display, you will observe the plot shown in Fig. 2 (once the system is turned on). But to take data you will use the dual trace mode and observe both the F-H signal and $V_a/10$ as a function of time.
3. Turn everything on, including the cathode heater, and allow the system to warm up and come to equilibrium for about twenty minutes. **Do not touch the hot oven!** Note the notation on the control unit: V_{ret} is labeled “reverse bias” and can be varied between 1 and 10 V. V_a is labeled “ V_a ” and can be varied from 0 to 70 V, but note that the output to the scope is $V_a/10$ and hence varies from 0 to 7 V. While the oven is heating observe the range over which the current changes as the rheostat is adjusted and observe the current setting where you can first see the heater glowing red. [If the resistance of the

Franck-Hertz Control Unit



Neva Franck-Hertz Oven and Triode



rheostat is set too large, the filament may never heat up enough to thermionically produce a detectable collector current.]

4. The mean free path needed to perform this experiment is obtained at around 185° C. When the oven temperature has stabilized near 185° C, set the scope to X-Y display and adjust the various voltages to obtain a display such as shown in Fig. 2. (Be sure the V_a selector switch is set on sawtooth.) Take time to understand how each of the variables affects the display. Note that when the heater current is changed, it takes several minutes for the pattern to stabilize.

5. Switch the display mode on the scope to DUAL. Note that the $V_a/10$ signal on the scope is not a linear ramp. Rather, you see the signal is a half wave rectified sine wave. So the F-H signal on the oscilloscope (channel 1), when observed in dual display with a time sweep for the x-axis, is not linear in V_a ; the peaks are not equally spaced. To record data, move the t1 cursor on the scope to the center of the peak (or valley) you want to measure. Then move the v2 cursor to the point where the t1 cursor intersects the $V_a/10$ signal. The v2 reading is $V_a/10$ for that peak or valley. The value of t1 is irrelevant.

6. The Franck-Hertz experiment claims to show that the energy levels of Hg are quantized, which implies that the spacing of the peaks (or valleys) do not depend on other variables like the retarding voltage, the filament current (temperature) or the oven temperature. To verify this select two adjacent peaks and the two associated valleys and measure V_a for three different values of the retarding voltage, three filament currents and three oven temperatures (~ 195 °C, 185 °C, and 170 °C). Calculate the peak and valley separation for each case and present your results in a table. [The different filament currents are intended to test whether the kinetic energy distribution of the thermionically emitted electrons affects the location of the peaks. What can you conclude from your data?]

7. With the temperature near 185 °C select a suitable value of V_{ret} and optimize the various settings to observe as many peaks as possible. Using the techniques outlined in step 5 measure the values of V_a for as many successive valleys and peaks as possible. [Note that your results from step 6 imply that you can change various parameters to more clearly see the peaks or valleys without affecting the data you took at previous settings.]

ANALYSIS: Assign an index number ($n_{col} = 1, 2, \text{etc.}$) to each of the peaks and the following valley. You may not have been able to see the first few peaks, but by using Eq. (5) you can deduce the correct value for n_{col} . Make a plot of V_a versus n for the peaks and valleys. The plot should give you two straight lines. For the peaks the equation for the line is:

$$V_a = V_{contact} - V_{thermal} + n_{col} V_{(1P-0S)} \quad (6)$$

while for the valleys:

$$V_a = V_{contact} - V_{thermal} + V_{max} + n_{col} V_{(1P-0S)}, \quad (7)$$

where V_{contact} is the contact potential, V_{thermal} is the kinetic energy the electrons have after thermionic emission by the cathode and V_{max} is the energy above threshold where the 3P_1 excitation reaches a maximum. The accepted value of $V_{(1^3P-1^1S)} = 4.86\text{V}$. Interpret the results of your fit in terms of these parameters. Estimate the fitting errors.

REPORT: In your report:

1. Give a table showing the peak and valley separations for different values of the retarding voltage, filament current and oven temperature. Do your data show any evidence that the initial kinetic energy of the thermionically emitted electrons varies with cathode temperature?
2. Interpret the slopes and intercepts and give values for the parameters used in Eqs. (6) and (7). What conclusions can you draw? Discuss the errors and the sources of error.
3. Discuss why the peaks and valleys of the collector current rounded.
4. Suppose the excitation probabilities of all three triplet excited states were about the same. How would the shape of the observed collector current curves change for "high" Hg vapor pressure ["high" = probability that an inelastic scattering occurs in a voltage interval much less than that between the threshold energy (the peak) and the energy for maximum excitation cross section (the valley).]
5. Use Eq. (1) plus the tabulated values of the temperature dependence of the vapor pressure of Hg to prepare a graph of the theoretical mean free path of electrons in mercury vapor versus temperature. (Note that this equation includes all types of collisions including elastic ones and thus is shorter than the mean free path for inelastic collisions that excite the atom to the 3P_1 state.) If this equation applied for inelastic collisions only, what would be the temperature range over which you would do the experiment?

READINGS:

1. R. Eisberg and R. Resnick: Quantum Physics of Atoms, Molecules, Solids, Nuclei and Particles, pp 107-110 (F-H effect in Hg), pp 407-409 (contact potential, thermionic emission)
2. D.W. Preston and E.R. Dietz: The Art of Experimental Physics, Experiment 6, pp 197-208
3. Neva: Franck-Hertz Experiment, KA6040/41; 6750-984 (manual)
4. D. Halliday and R. Resnick: Fundamentals of Physics, New York, John Wiley, 3rd edition 1988: pp. 491-492.