Error Analysis

Measurement of one quantity

Let {x1, x2, x3, …., xN} be the results of N measurements of a certain quantity.

Mean: $\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$

Standard deviation: $\sigma_x = \sqrt{\frac{1}{N-1}\sum\left(x_i - \bar{x}\right)^2}$

Standard deviation of the mean: $\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{N}}$ (“standard error”)

The final result of the measurements may be given as:

Value of x $= \bar{x} \pm \frac{\sigma_x}{\sqrt{N}}$

Least square fit to straight line

Let {(x1,y1), (x2,y2), ….(xN,yN)} be pairs of N measurements. The least-square straight line fit of these N data points in the form, y = A + Bx, is given by:

$$A = \frac{\left(\sum x_i^2\right)\left(\sum y_i\right) - \left(\sum x_i\right)\left(\sum x_i y_i\right)}{\Delta}$$

$$B = \frac{N\left(\sum x_i y_i\right) - \left(\sum x_i\right)\left(\sum y_i\right)}{\Delta}$$

where

$$\Delta = N\left(\sum x_i^2\right) - \left(\sum x_i\right)^2.$$

The standard deviation in y is given by:

$$\sigma_y = \sqrt{\frac{1}{N-2}\sum_{i=1}^{N}\left(y_i - A - Bx_i\right)^2}.$$

Uncertainties in A and B:

$$\sigma_A = \sigma_y\sqrt{\frac{\sum x_i^2}{\Delta}}$$

$$\sigma_B = \sigma_y\sqrt{\frac{N}{\Delta}}$$

Correlation coefficient (“goodness to fit”) is given by:

$$r = \frac{\sigma_{xy}}{\sigma_x\sigma_y} = \frac{\sum\left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right)}{\sqrt{\sum\left(x_i - \bar{x}\right)^2 \sum\left(y_i - \bar{y}\right)^2}}.$$ Fit is good if r ~ 1.

The uncertainties in A and B and r are often calculated by graphing softwares.

Poisson Distribution

In counting radioactive decays and some other similar random processes, the probability of *n* counts (during some definite time interval) is

$$P(\text{n counts}) \equiv P_{\mu}(n) = e^{-\mu}\frac{\mu^{n}}{n!}$$

where μ is the expected average count during the time interval above,

$$\bar{n} = \mu\ .$$

The standard deviation σ is:

$$\sigma = \sqrt{\mu}\ .$$