1)

**Problem 9.15**

Equation 9.78 is replaced by \( \vec{E}_{0r} \hat{x} + \vec{E}_{0r} \hat{u}_R = \vec{E}_{0r} \hat{u}_T \), and Eq. 9.80 becomes \( \vec{E}_{0r} \hat{y} - \vec{E}_{0r} (\hat{x} \times \hat{u}_T) = \beta \vec{E}_{0r} (\hat{x} \times \hat{u}_T) \). The \( y \) component of the first equation is \( \vec{E}_{0r} \sin \theta_R = \vec{E}_{0r} \sin \theta_T \); the \( x \) component of the second is \( \vec{E}_{0r} \sin \theta_R = - \beta \vec{E}_{0r} \sin \theta_T \). Comparing these two, we conclude that \( \sin \theta_R = \sin \theta_T = 0 \), and hence \( \theta_R = \theta_T = 0 \).  qed

2)

**Problem 9.19**

(a) Equation 9.120 \( \Rightarrow \tau = \epsilon/\sigma \). Now \( \epsilon = \epsilon_0 \varepsilon_r \) (Eq. 4.34), \( \varepsilon_r \approx \eta^2 \) (Eq. 9.70), and for glass the index of refraction is typically around 1.5, so \( \epsilon \approx (1.5)^2 \times 8.85 \times 10^{-12} = 2 \times 10^{-11} \text{ C}^2/\text{Nm}^2 \), while \( \sigma = 1/\rho \approx 10^{-12} (\Omega \text{m})^{-1} \) (Table 7.1). Then \( \tau = (2 \times 10^{-11})/10^{-12} = 20 \text{ s.} \) (But the resistivity of glass varies enormously from one type to another, so this answer could be off by a factor of 100 in either direction.)

(b) For silver, \( \rho = 1.59 \times 10^{-8} \) (Table 7.1), and \( \varepsilon \approx \epsilon_0 \), so \( \omega \varepsilon = 2 \pi \times 10^{10} \times 8.85 \times 10^{-12} = 0.56 \).

Since \( \sigma = 1/\rho = 6.25 \times 10^7 \gg \omega \varepsilon \), the skin depth (Eq. 9.128) is

\[
    d = \frac{1}{\kappa} \approx \sqrt{\frac{2}{\omega \sigma \mu}} = \sqrt{\frac{2}{2 \pi \times 10^{10} \times 6.25 \times 10^7 \times 4\pi \times 10^{-7}}} = 6.4 \times 10^{-7} \text{ m} = 6.4 \times 10^{-4} \text{ mm.}
\]

I’d plate silver to a depth of about \( 0.001 \text{ mm;} \) there’s no point in making it any thicker, since the fields don’t penetrate much beyond this anyway.

(c) For copper, Table 7.1 gives \( \sigma = 1/(1.68 \times 10^{-8}) = 6 \times 10^7 \), \( \omega \varepsilon_0 = (2 \pi \times 10^6) \times (8.85 \times 10^{-12}) = 6 \times 10^{-5} \).

Since \( \sigma \gg \omega \varepsilon \), Eq. 9.126 \( \Rightarrow k \approx \sqrt{\frac{\omega \sigma \mu}{2} \text{, so (Eq. 9.129)}} \)

\[
    \lambda = 2\pi \sqrt{\frac{2}{\omega \sigma \mu_0}} = 2\pi \sqrt{\frac{2}{2 \pi \times 10^6 \times 6 \times 10^7 \times 4\pi \times 10^{-7}}} = 4 \times 10^{-4} \text{ m} = 0.4 \text{ mm.}
\]

From Eq. 9.129, the propagation speed is \( v = \frac{\omega}{k} = \frac{\omega}{2\pi \lambda} = \lambda \nu = (4 \times 10^{-4}) \times 10^6 = 400 \text{ m/s.} \) In vacuum

\[
    \frac{\lambda}{\nu} = \frac{3 \times 10^8}{10^6} = 300 \text{ m/s;} \quad \nu = c = \frac{3 \times 10^8 \text{ m/s.}}{(\text{But really, in a good conductor the skin depth is so small compared to the wavelength, that the notions of “wavelength” and “propagation speed” lose their meaning.})
3) Problem 9.20

(a) Use the binomial expansion for the square root in Eq. 9.126:

\[ \kappa \equiv \omega \sqrt{\frac{\epsilon \mu}{2}} \left[ 1 + \frac{1}{2} \left( \frac{\sigma}{\epsilon \omega} \right)^2 - 1 \right]^{1/2} = \omega \sqrt{\frac{\epsilon \mu}{2}} \frac{1}{\sqrt{2}} \frac{\sigma}{\epsilon \omega} = \frac{\sigma}{2} \sqrt{\frac{\mu}{\epsilon}}. \]

So (Eq. 9.128) \( d = \frac{1}{\kappa} \approx \frac{2}{\sigma} \sqrt{\frac{\epsilon}{\mu}} \). \( \text{qed} \)

For pure water, \( \epsilon_r = 80.1 \) (Table 6.1) and \( \sigma = 1/(8.3 \times 10^3) \) (Table 7.1) so

\[ d = (2)(8.3 \times 10^3) \sqrt{\frac{(80.1)(8.85 \times 10^{-12})}{4\pi \times 10^{-7}}} = 382 \text{ m} \]

For sea water \( \sigma = 1/(0.2) \) and \( d \approx 1 \text{ cm} \).

(b) In this case \((\sigma/\epsilon \omega)^2\) dominates, so (Eq. 9.126) \( k \approx \kappa \), and hence (Eqs. 9.128 and 9.129)

\[ \lambda = \frac{2\pi}{k} \approx \frac{2\pi}{\kappa} = 2\pi d, \text{ or } d = \frac{\lambda}{2\pi}. \text{ qed} \]

Meanwhile \( \kappa \approx \omega \sqrt{\frac{\epsilon \mu}{2}} \sqrt{\frac{\sigma}{\epsilon \omega}} = \sqrt{\frac{\omega \mu \sigma}{2}} = \sqrt{\frac{(10^{15})(4\pi \times 10^{-7})(10^7)}{2}} = 8 \times 10^7; \quad d = \frac{1}{\kappa} = \frac{1}{8 \times 10^7} = 1.3 \times 10^{-8} = [13 \text{ nm}]. \) So the fields do not penetrate far into a metal—which is what accounts for their opacity.

(c) Since \( k \approx \kappa \), as we found in (b), Eq. 9.134 says \( \phi = \tan^{-1}(1) = 45^\circ \). \( \text{qed} \)

Meanwhile, Eq. 9.137 says \( \frac{B_0}{E_0} \approx \sqrt{\frac{\epsilon \mu}{\epsilon \omega}} = \sqrt{\frac{\sigma u}{\omega}} \). For a typical metal, then, \( \frac{B_0}{E_0} = \sqrt{\frac{(10^7)(4\pi \times 10^{-7})}{10^{15}}} = \frac{10^{-7}}{\text{s/m}} \) (In vacuum, the ratio is \( 1/c = 1/(3 \times 10^8) = 3 \times 10^{-9} \text{s/m}, so the magnetic field is comparatively about 100 times larger in a metal.)

4) Problem 9.22

According to Eq. 9.147, \( R = \left| \frac{E_{0R}}{E_{0I}} \right|^2 = \left| \frac{1-i\beta}{1+i\beta} \right|^2 = \left( \frac{1-i\beta^*}{1+i\beta^*} \right), \text{ where } \beta = \frac{\mu_1 v_1}{\mu_2 \omega} (k_2 + i\kappa_2) \) (Eqs. 9.125 and 9.146). Since silver is a good conductor \((\sigma \gg \epsilon \omega)\), Eq. 9.126 reduces to

\[ \kappa_2 \approx k_2 \approx \omega \sqrt{\frac{\epsilon_2 \mu_2}{2}} \sqrt{\frac{\sigma}{\epsilon_2 \omega}} = \sqrt{\frac{\sigma \mu_2}{2 \omega}}, \text{ so } \beta = \frac{\mu_1 v_1}{\mu_2 \omega} \sqrt{\frac{\sigma \mu_2}{2 \omega}} \sqrt{ \frac{(1 + i)}{2} } \frac{\sigma}{\mu_2 \omega} = (3 \times 10^8) \sqrt{\frac{(6 \times 10^7)(4\pi \times 10^{-7})}{2}} = 29. \text{ Then} \]

\[ R = \left( \frac{1-i\gamma}{1+i\gamma} \right) \left( \frac{1-i\gamma^*}{1+i\gamma^*} \right) = \left( \frac{1-\gamma^2+\gamma^2}{(1+\gamma)^2+\gamma^2} \right) = 0.93. \] Evidently 93% of the light is reflected.