1) Problem 7.7

(a) \( E = - \frac{d\Phi}{dt} = -Bl^2 \frac{dx}{dt} = -Blv; \ E = IR \Rightarrow I = \frac{Blv}{R} \). (Never mind the minus sign—it just tells you the direction of flow: \( \mathbf{v} \times \mathbf{B} \) is upward, in the bar, so downward through the resistor.)

(b) \( F = IlB = \frac{B^2l^2v}{R} \), to the left.

(c) \( F = ma = m \frac{dv}{dt} = -\frac{B^2l^2}{R} v \Rightarrow \frac{dv}{dt} = -\left( \frac{B^2l^2}{Rm} \right) v \Rightarrow v = v_0 e^{-\frac{B^2l^2 t}{Rm}}. \)

(d) The energy goes into heat in the resistor. The power delivered to resistor is \( I^2R \), so

\[
\frac{dW}{dt} = I^2R = \frac{B^2l^2v^2}{R^2} R = \frac{B^2l^2}{Rm} v_0^2 e^{-2\alpha t}, \quad \text{where} \quad \alpha = \frac{B^2l^2}{Rm}; \quad \frac{dW}{dt} = \alpha mv_0^2 e^{-2\alpha t}.
\]

The total energy delivered to the resistor is

\[
W = \alpha mv_0^2 \int_0^\infty e^{-2\alpha t} dt = \alpha mv_0^2 \left[ \frac{e^{-2\alpha t}}{-2\alpha} \right]_0^\infty = \alpha mv_0^2 \frac{1}{2\alpha} = \frac{1}{2} mv_0^2. \quad \checkmark
\]

2) Slightly simplified version of 7.11

Notation: \( \sigma = \text{conductivity}, \ \rho = V = \text{resistivity}, \ \eta = \text{mass density of aluminum}; \ V = 4lwd = \text{volume of material in the loop}. \) So the downward gravitational force is \( F_g = g\eta V \). Now \( E = Blv = IR \) so \( I = Blv/R \) and upward magnetic force is \( F_m = Ilb = B^2l^2v/R \). But \( R = 4l/wd\sigma \) so \( F_m = B^2lvd\sigma/4 = B^2vV/16\sigma \). For the terminal velocity, equate \( F_m = F_g \) (V’s cancel!) to get \( v_t = \frac{16\eta g\rho}{B^2} \). Plug in numbers: \( \rho = 2.8 \times 10^{-8} \Omega m, \ g = 98 \text{m/s}^2, \ \eta = 2.7 \times 10^3 \text{kg/m}^3, \ B = 1 \text{T}; \ v_t = 1.2 \text{cm/s}. \)

3) Problem 7.12

\[
\Phi = \pi \left( \frac{a}{2} \right)^2 B = \frac{\pi a^2}{4} B_0 \cos(\omega t); \ E = -\frac{d\Phi}{dt} = \frac{\pi a^2}{4} B_0 \omega \sin(\omega t). \quad I(t) = \frac{E}{R} = \frac{\pi a^2 \omega}{4R} B_0 \sin(\omega t).
\]

4) Problem 7.16

(a) The magnetic field (in the quasistatic approximation) is “circumferential”. This is analogous to the current in a solenoid, and hence the field is \( \text{longitudinal}. \)

(b) Use the “amperian loop” shown.

Outside, \( B = 0 \), so here \( E = 0 \) (like \( B \) outside a solenoid). So \( \oint \mathbf{E} \cdot d\ell = E \ell = -\frac{d\Phi}{dt} = -\frac{d}{dt} \int_s \mathbf{B} \cdot d\mathbf{a} = -\frac{d}{dt} \int_s \frac{\mu_0 I}{2\pi s} ds' \).

\[
\therefore \ E = -\frac{\mu_0 I}{2\pi} \frac{d}{dt} \ln \left( \frac{a}{s} \right). \quad \text{But} \quad \frac{d}{dt} = -I_0 \omega \sin \omega t,
\]

so

\[
E = \frac{\mu_0 I_0 \omega}{2\pi} \sin(\omega t) \ln \left( \frac{a}{s} \right) \dot{.}
\]
Problem 7.17

(a) The field inside the solenoid is $B = \mu_0 n I$. So $\Phi = \pi a^2 \mu_0 n I \Rightarrow \mathcal{E} = -\pi a^2 \mu_0 n (dI/dt)$.

In magnitude, then, $\mathcal{E} = \pi a^2 \mu_0 n k$. Now $\mathcal{E} = I_r R$, so $I_{\text{resistor}} = \frac{\pi a^2 \mu_0 n k}{R}$.

$\mathbf{B}$ is to the right and increasing, so the field of the loop is to the left, so the current is counterclockwise, or to the right, through the resistor.

(b) $\Delta \Phi = \pi a^2 \mu_0 n I$; $I = \frac{dQ}{dt} = \frac{\mathcal{E}}{R} = -\frac{1}{R} \frac{d\Phi}{dt} \Rightarrow \Delta Q = \frac{1}{R} \Delta \Phi$, in magnitude. So $\Delta Q = \frac{\pi a^2 \mu_0 n I}{R}$. 