1) \( R = \frac{R_1 R_2}{R_1 + R_2} \) (Resistors in Parallel)

\( C = C_1 + C_2 \) (Capacitors in Parallel)

\[ Q(t) = CV(t) = RC \int i(t) dt = -RC \frac{dQ}{dt} \]

\[ \Rightarrow Q(t) = Q_0 e^{-\frac{t}{RC}} \]

\[ V(t) = V_0 e^{-\frac{t}{RC}} \]

\[ T = RC = \frac{R_1 R_2 (C_1 + C_2)}{R_1 + R_2} \]

(b) \( Q_1(t) = C_1 V(t) = \frac{V_0}{R_1} e^{-\frac{t}{RC}} \)

(c) \( \dot{I}_1(t) = \frac{1}{R_1} V(t) = \frac{V_0}{R_1} e^{-\frac{t}{RC}} \)

2) Problem 7.1

(a) Let \( Q \) be the charge on the inner shell. Then \( E = \frac{1}{4\pi \varepsilon_0} \frac{Q}{r^2} \) in the space between them, and \( (V_a - V_b) = -\int_b^a E \cdot dr = -\frac{1}{4\pi \varepsilon_0} Q \int_b^a \frac{1}{r^2} dr = \frac{Q}{4\pi \varepsilon_0} \left( \frac{1}{a} - \frac{1}{b} \right) \).

\[ I = \int j \cdot da = \sigma \int E \cdot da = \frac{\sigma}{\varepsilon_0} \frac{4\pi \varepsilon_0 (V_a - V_b)}{(1/a - 1/b)} = \frac{4\pi \sigma}{\varepsilon_0} \frac{(V_a - V_b)}{(1/a - 1/b)} \]

(b) \( R = \frac{V_a - V_b}{I} = \frac{1}{4\pi \sigma} \left( \frac{1}{a} - \frac{1}{b} \right) \).

(c) For large \( b (b \gg a) \), the second term is negligible, and \( R \approx 1/4\pi \sigma a \). Essentially all of the resistance is in the region right around the inner sphere. Successive shells, as you go out, contribute less and less, because the cross-sectional area \( (4\pi r^2) \) gets larger and larger. For the two submerged spheres, \( R = \frac{2}{4\pi \sigma a} = \frac{1}{2\pi \sigma a} \) (one \( R \) as the current leaves the first, one \( R \) as it converges on the second). Therefore \( I = \frac{V}{R} = \frac{1}{2\pi \sigma a V} \).

(d) Energy from battery: \( \int_0^\infty V_0 I dt = \int_0^\infty \frac{V_0^2}{R} e^{-t/RC} dt = \frac{V_0^2}{R} \left( \frac{RC}{RC} \right) \mid_0^\infty = \frac{V_0^2}{R} RC = CV_0^2 \).

Since \( I(t) \) is the same as in (a), the energy delivered to the resistor is again \( \frac{1}{2} CV_0^2 \). The final energy in the capacitor is also \( \frac{1}{2} CV_0^2 \), so half the energy from the battery goes to the capacitor, and the other half to the resistor.
3) The solution with \( \nabla \cdot \mathbf{J} = \sigma \nabla \times \mathbf{E} = \sigma \frac{\Phi}{4\pi \varepsilon_0} \frac{\mathbf{n}}{r^2} \) solves \( \nabla \cdot \mathbf{J} = 0 \) as before, and \( \mathbf{n} \cdot \mathbf{J} = 0 \) for the bottom surface, so the same solution matches the new boundary conditions.

4) \[
V = \int_1^{2l} E_x \, dx = \int_1^{2l} \frac{T}{A} \mathbf{J} \, dx = \frac{I}{A} \int_1^{2l} \frac{1}{\sigma(x)} \, dx
\]

\[
= \frac{I}{A} \int_1^{2l} \frac{1}{\sigma_0} \ln \frac{L}{x} \, dx = \frac{I L}{A \sigma_0} \left[ \ln (2L) - \ln L \right] = IR
\]

\[
R = \frac{L \ln (2)}{\sigma_0 A}
\]

(You might have guessed that the average conductivity is \( \frac{3}{2} \sigma_0 \). Instead, it acts like it has a conductivity of \( 5.0 \ln 2 \approx 1.44 \sigma_0 \), not \( 1.5 \sigma_0 \).)