Due date: April 17

Reading: Chapter 10

1. [4 points] Griffiths 10.3 (Fields, charges and currents from $V$ and $A$ and gauge change).

2. [3 points] Consider the potentials $V(r,t) = 0$ and $A(r,t) = A_0 \hat{y} \sin(kx) \sin(\omega t)$ (representing “standing electromagnetic waves”).
   
   (a) Show that these potentials obey the Lorentz gauge condition.
   
   (b) Show that these potentials satisfy the usual equations of motion for the potentials in vacuum, $\nabla^2 A = \frac{1}{c^2} \frac{\partial^2 A}{\partial t^2}$ and $\nabla^2 V = \frac{1}{c^2} \frac{\partial^2 V}{\partial t^2}$, provided that $k = \omega/c$.
   
   (c) Find $E(r,t)$ and $B(r,t)$.


4. [6 points] Suppose it happens that $J(r,t) = J(r)$ and $\rho(r,t) = \gamma(r)t$ (that is, the current is constant in time and the density is linear in time).
   
   (a) Show that conservation of total charge $Q = \int \rho \, d\tau$ implies that $\int \gamma(r') \, d\tau' = 0$.
   
   (b) Write the equations for the retarded potentials in terms of $\rho$ and $J$ and use these to show that $A$ is constant in time and $V$ is linear in time, that is, $V(r,t) = w(r)t$ and $A(r,t) = A(r)$, where
   
   $$w(r) = \frac{1}{4\pi \epsilon_0} \int \frac{\gamma(r')}{z} \, d\tau'$$

   and
   
   $$A(r) = \frac{\mu_0}{4\pi} \int \frac{J(r')}{z} \, d\tau'.$$

   **Hint:** One of these is trivial. For the other you have to notice a fortuitous cancellation of the $z$ in the numerator and denominator of one contribution, and use the result of part (a).

   (c) Using the potentials-to-fields equations, show furthermore that $B$ is constant in time and $E$ is linear in time, with $E(r,t) = -\nabla w(r)t$ and $B(r,t) = \nabla \times A(r)$.

5. [4 points] Griffiths 10.20 ($E$ and $B$ on $x$-axis for charge on $x$-axis).