Ground rules:

- Open book, open lecture notes
- You are expected to use a calculator to answer some questions
- Correct treatment of units, factors of $2\pi$, etc. will count in grading these problems
- Write your answer directly on these sheets (continue onto back, if necessary)

There are four questions of 25 points each. Pace yourself accordingly.

If you know the formulas for “standard cases,” you may use these results without derivation unless the problem specifically asks you to derive it.

Partial credit will be given. Do as many parts of a problem as possible. If you are stuck on the first part of a problem, you should still try to say something about the later parts if possible.

Be sure to check that you have done all parts of all questions.

Feel free to raise your hand to ask a question.

Good luck!!

You might or might not need these on the exam:

$$\epsilon_0 = 8.85 \times 10^{-12} \text{C}^2/\text{Nm}^2, \quad \mu_0 = 4\pi \times 10^7 \text{C}^2\text{N}/\text{A}^2$$

$$\sqrt{\frac{\mu_0}{\epsilon_0}} = 377 \Omega$$

$$\frac{1}{\sqrt{\mu_0\epsilon_0}} = c = 3 \times 10^8 \text{m/s}$$
Problem 1 (25 points)
Consider an electromagnetic wave in vacuum (with $\omega = ck$) whose electric field part is

$$\mathbf{E}(r, t) = E_0 \hat{x} \cos(ky + \omega t + \varphi_0)$$

(a) Describe the direction of propagation and the polarization of the wave.

(b) Write down the corresponding $\mathbf{B}$-field.

(c) Find the energy density $u_{em}(r, t)$ and the Poynting vector $\mathbf{S}(r, t)$, eliminating $\mu_0$ from your answers and writing them in terms of $\epsilon_0$ and $c$ instead.

Note: For (b-c) you can continue to express your answers as a function of $(ky + \omega t + \varphi_0)$.

(d) Starting from $t=0$, what is the first time a node of the EM wave passes the origin, if $\varphi_0 = \pi/6$?
**Problem 2** (25 points)

The region to the left of the plane \( x = 0 \) is vacuum, and the region to the right contains a dispersive medium with dispersion relation \( k(\omega) = \alpha \omega^{3/2} \), where \( \alpha \) is some constant.

(a) Find the index-of-refraction function \( n(\omega) \) of the dispersive medium.

(b) White light is incident from the vacuum side with incident angle \( \theta_I = 60^\circ \). As you know, white light is made up of components of many frequencies. If the component with frequency \( \omega_0 \) is found to exit with transmitted angle \( \theta_T = 30^\circ \), then what will be the transmitted angle for the component at frequency \( \omega_0/2 \) ?
Problem 3 (25 points)

Consider a “poor conductor” with conductivity $\sigma = 4.8 \, (\Omega\cdot m)^{-1}$, and free-space values of $\epsilon$ and $\mu$. (The three parts of this question are independent; do as many as you can. Formulas on the cover page may help. Answers should be numerical, with units.)

(a) Calculate the relaxation time $\tau$, and say briefly what its physical meaning is.

(b) For a poor conductor, the skin depth is well approximated by $d = (2/\sigma)\sqrt{\epsilon/\mu}$. Calculate $d$, and say briefly what its physical meaning is.

(c) At a certain frequency, the skin depth is 10 times the wavelength inside this material. Find the phase lag $\phi$ between the electric and magnetic fields, in degrees.
Problem 4 (25 points)

A rectangular waveguide has width $3d$ and height $d$ in the $x$ and $y$ directions respectively. For parts (a-d), just express your answers in terms of $d$.

(a) Find the cutoff frequency $\omega_{30}$ of the TE$_{30}$ mode.
(b) Find the phase and group velocities for a wave propagating down the waveguide in the TE$_{30}$ mode at twice that frequency, i.e., $2\omega_{30}$.
(c) Is there another TE mode having the same cutoff frequency as the TE$_{30}$ mode? If so, what is it?
(d) Are there any TM modes that can propagate at frequency $\omega_{30}$? If so, which?
(e) If $d = 1.2$ cm, what is the cutoff frequency $\nu_{30}$ of the TE$_{30}$ mode, expressed in MHz?