Ground rules:

- Open book
- Closed notes
- You may consult one page (two sides) of handwritten notes
- Also OK to consult handout on conservation laws
- A calculator is allowed but will not be useful
- Write your answer directly on these sheets (continue onto back, if necessary)

There are four questions of 25 points each. Pace yourself accordingly.

If you know the formulas for “standard cases” (e.g., the electric field or potential a certain distance from a point or line or plane charge), you may use these results without derivation unless the problem specifically asks you to derive it.

Partial credit will be given. Do as many parts of a problem as possible. If you are stuck on the first part of a problem, you should still try to say something about the later parts if possible.

Be sure to check that you have done all parts of all questions.

Feel free to raise your hand to ask a question.

Good luck!!
Problem 1. (25 points)

Two wire loops lie next to each other as shown. The only things you know are that the resistance of the circular loop is $R_2$; the mutual inductance of the two loops is $M$; and the current in the square loop is some given $I_1(t)$.

(a) Suppose $I_1(t) = I_0$, $t < 0$
$I_1(t) = I_0 \left(1 - \frac{t^2}{\tau^2}\right)$, $0 < t < \tau$
$I_1(t) = 0$, $t > \tau$

(For example, perhaps the square loop had been connected to a battery until time $t = 0$, when some switch was opened over a time interval $\tau$.) Find the current $I_2(t)$ induced in the circular loop.

(b) Find the total charge $Q$ that flows in the circular loop during this time.

(c) Suppose all you know is that $I_1(t)$ falls monotonically from $I_0$ to 0 during some time. Is the answer to part (b) still the same? If so, show why.
Problem 2. (25 points)

Here is a useful contactless way of measuring the resistance of a coin. A solenoid (large cylinder with its axis along the $z$ axis) is driven with an oscillating current through the windings in such a way that the magnetic field inside is $\mathbf{B}(t) = B_0 \cos(\omega t) \hat{z}$.

(a) Find $\mathbf{E}(s)$ (magnitude and direction) inside the solenoid ($s$ is the distance from the axis).

(b) A coin is placed at the center of the solenoid as shown. We model it as a disk of radius $R$, thickness $d$, and conductivity $\sigma$. Find $\mathbf{J}(s)$ inside the disk.

(c) Find the total power $P(t)$ being dissipated to Joule heating inside the disk at instant $t$. (Hint: an integral is required.)

(d) Summarize by giving an expression for the average power dissipated, $\langle P \rangle$, in terms of $\sigma$, $B_0$, $\omega$, $R$, and $d$. (That is, $\langle P \rangle$ is the average of $P(t)$ over one cycle of the oscillation.)
Problem 3. (25 points)

Suppose the electric field is known to be

\[ \mathbf{E}(\mathbf{r}, t) = \frac{\alpha}{s} \sin(\omega t) \hat{z} \]

independent of \( z \) and \( \phi \), where \( \alpha \) is a constant, and \( s \) is the radial coordinate of the \((s, \phi, z)\) cylindrical coordinate system. No actual currents flow; that is, \( \mathbf{J} = 0 \).

(a) Find the displacement current \( \mathbf{J}_d(\mathbf{r}, t) \).

(b) Find the magnetic field induced by the displacement current at time \( t = 0 \).

Hint: Argue that \( \mathbf{B} = B(s)\hat{\phi} \) (why?) and compute \( B(s) \).
Problem 4. (25 points)

Consider two oppositely charged sheets, separated by a distance \( d \) as shown; both sheets are moving to the right at the same velocity \( \mathbf{v} = v \hat{x} \).

(a) Find \( \mathbf{E} \) and \( \mathbf{B} \) between the plates.

(b) Find the electromagnetic energy density \( u_{em} \) between the plates. Show that if \( v \ll c \) then the magnetic contribution is negligible.

(c) Compute the Poynting vector \( \mathbf{S} \) between the plates.

(d) In the approximation \( v \ll c \), compute the “apparent velocity” \( v_{\text{app}} = \mathbf{S} / u_{em} \) of the energy density between the plates.

(Hint: The answer to part (d) is a bit peculiar...)