Ground rules:

- Open book
- Closed notes
- You may consult one page (two sides) of handwritten notes
- Write your answer directly on these sheets (continue onto back, if necessary)

There are four questions of 25 points each. Pace yourself accordingly.

If you know the formulas for “standard cases” (e.g., the electric field or potential a certain distance from a point or line or plane charge), you may use these results without derivation unless the problem specifically asks you to derive it.

Partial credit will be given. Do as many parts of a problem as possible. If you are stuck on the first part of a problem, you should still try to say something about the later parts if possible.

Be sure to check that you have done all parts of all questions.

Feel free to raise your hand to ask a question.

Good luck!!
**Problem 1.** (25 points)

A magnetic field of strength $B$ is pointing toward you out of the plane of the paper. The U-shaped metal rail is resistanceless, but a resistive metal bar (resistance $R$ between the two points of contact a distance $d$ apart) slides along the rails to the right at speed $v$.

(a) What current flows through the bar, and in what direction?

(b) What is the magnetic force on the bar, and in which direction?

(c) What is the power being dissipated to resistance in the bar? Does this match the mechanical power being applied to move the bar?
Problem 2. (25 points)

Here is a useful contactless way of measuring the resistance of a coin. A solenoid (large cylinder with its axis along the $z$ axis) is driven with an oscillating current through the windings in such a way that the magnetic field inside is $B(t) = B_0 \cos(\omega t) \hat{z}$.

(a) Find $E(s)$ (magnitude and direction) inside the solenoid ($s$ is the distance from the axis).

(b) A coin is placed at the center of the solenoid as shown. We model it as a disk of radius $R$, thickness $d$, and conductivity $\sigma$. Find $J(s)$ inside the disk.

(c) Find the total power $P(t)$ being dissipated to Joule heating inside the disk at instant $t$. (Hint: An integral is required. Recall that the energy lost to resistive heating, per unit volume per unit time, is $E \cdot J$.)

(d) Summarize by giving an expression for the average power dissipated, $\langle P \rangle$, in terms of $\sigma$, $B_0$, $\omega$, $R$, and $d$. (That is, $\langle P \rangle$ is the average of $P(t)$ over one cycle of the oscillation.)
Problem 3. (25 points)

A parallel-plate capacitor has circular plates of radius $R$ a distance $d$ apart (neglect fringing fields), and is fed by wires carrying a current $I(t) = \gamma t^2$ where $\gamma$ is a constant. The charge on the plates is zero at $t = 0$.

(a) Find the charge $Q(t)$ on the bottom plate.
(b) Find the electric field $E(t)$ between the plates.
(c) Find the displacement current $J_d(t)$ between the plates.
(d) Find the induced magnetic field $B(s, t)$ (magnitude and direction) for $s < R$ between the plates, neglecting fringing-field effects ($s$ is the distance from the axis).
Problem 4. (25 points)

An infinitely long cylinder of radius \( R \) is centered on the \( z \) axis and contains a magnetic field that grows with time according to

\[
B(s, t) = \begin{cases} 
\alpha s^2 e^{t/\tau} \hat{z} & \text{for } s < R \\
0 & \text{otherwise}
\end{cases}
\]

where \( \alpha \) and \( \tau \) are constants, \( s \) is distance from the axis, and \( t \) is time. No charges are present. As a function of \( t \):

(a) Find the resulting electric field (magnitude and direction) a distance \( s \) from the axis, both for \( s < R \) and \( s > R \).

(b) Find the Poynting vector \( S \) a distance \( s \) from the axis, both for \( s < R \) and \( s > R \).