Ground rules:

- Open book
- Open notes
- You are expected to use a calculator to answer some questions
- Write your answer directly on these sheets (continue onto back, if necessary)

There will be 8 questions for a total of 100 points on the actual exam.

In this practice exam there are only three questions. This is because I am only trying to cover the material here from after Hour Exam 2. Use the questions on Hour Exams 1 and 2, and their corresponding Practice Exams, and the three Quizzes, as examples of what the other questions will look like. All are posted on the website.

Show your reasoning. A correct answer all by itself, without any indication of the reasoning leading to it, will not get full credit. However, you may use the formulas for “standard cases” (for example, the magnetic field a certain distance from a line current) in your solution without deriving them.

Partial credit will be given. Do as many parts of a problem as possible. If you are stuck on the first part of a problem, you should still try to say something about the later parts if possible.

Be sure to check that you have done all parts of all questions.

Feel free to raise your hand to ask a question.

Good luck!!
Problem 1 (10 points)

*Important:* If you have trouble with parts (a) and (b), assume the result quoted in part (b) and go on to parts (c) and (d).

A neutral current-carrying sheet lies in the $x$-$y$ plane and carries a uniform surface current density that was “turned on” at time $t = 0$: $\mathbf{K}(t) = 0$ for $t < 0$ and $\mathbf{K}(t) = K_0 \hat{x}$ for $t > 0$.

(a) For the target point $(0, 0, z)$ a distance $z$ above the plane, at target time $t > 0$, show that the retarded time is positive ($t_r > 0$) at source points in the $x$-$y$ plane for which $r < \sqrt{c^2 t^2 - z^2}$ (where $r = \sqrt{x^2 + y^2}$).

(b) Show that the retarded vector potential is

$$A_x(z, t) = \begin{cases} 0, & t < z/c \\ \left(\mu_0 K_0 / 2\right) (ct - z), & t > z/c \end{cases}$$

*Hint:* You should encounter an area integral of the form $\int_0^{\sqrt{c^2 t^2 - z^2}} (\text{integrand}) 2\pi r \, dr$, which can be evaluated via the substitution $u = z^2 + r^2$.

(c) Also find $A_y(r, t)$, $A_z(r, t)$, and $V(r, t)$. (Remember, the sheet is neutral.)

(d) Find $\mathbf{E}(z, t)$ and $\mathbf{B}(z, t)$. 
Problem 2 (10 points)

An electric dipole antenna is oriented along the $z$ axis. The maximum electric dipole moment is $p_0$, the angular frequency is $\omega$, and the total power being radiated is $P$.

(a) Starting from the standard expression for the power (no derivation needed), solve for $p_0$ in terms of $P$ and other quantities.

(b) Rewrite the standard formula for the intensity of radiation $I(r, \theta)$ far from the source, expressing your answer in terms of $P$ (i.e., eliminating $\omega$).

(c) The total power being radiated is $P = 10$ kW. What is the intensity of electromagnetic radiation incident on a receiving dish 1 km away (at the same $z$ elevation)?

(d) What is the direction of polarization of the EM wave incident on the receiving dish?

(e) How would your answers to (c) and (d) change, if at all, if it were instead a vertical magnetic dipole antenna ($\mathbf{m} \parallel \hat{z}$) radiating the same total power of 10 kW?
Problem 3 (15 points)

In inertial frame $S$, the $x$-$y$ plane is filled by a planar charge density $\sigma$ that is moving in the $+\hat{x}$ direction at speed $v$. The frame in which the charged sheet is at rest is called frame $S'$; that is, frame $S'$ also moves at velocity $+v\hat{x}$ relative to frame $S$. (Note: $\sigma$ is the surface charge as measured in frame $S$, not in frame $S'$.) We are only interested in the $E$ and $B$ fields above the plane ($z > 0$).

(a) Warm-up: the fields in frame $S$ are $E_z = \sigma/2\epsilon_0$, $B_y = -\mu_0\sigma v/2$, and other components zero. Working entirely in frame $S$, explain briefly why $E_z = \sigma/2\epsilon_0$, $B_y = -\mu_0\sigma v/2$, and other components are zero.

(b) By transforming the fields, find the components of $E'$ and $B'$ in frame $S'$. Simplify your results by eliminating $\epsilon_0$ in favor of $\mu_0$ or vice versa. Also note that $\gamma(1 - v^2/c^2) = 1/\gamma$.

(c) Now check your result by doing the problem the other way: find the charge density $\sigma'$ seen by an observer in $S'$, and then compute the fields that you expect from the sources by working entirely in $S'$. 