**Problem 1** (10 points)

**Important:** If you have trouble with parts (a) and (b), assume the result quoted in part (b) and go on to parts (c) and (d).

A neutral current-carrying sheet lies in the \( x-y \) plane and carries a uniform surface current density that was “turned on” at time \( t = 0 \): \( K(t) = 0 \) for \( t < 0 \) and \( K(t) = K_0 \hat{x} \) for \( t > 0 \).

(a) For the target point \((0,0,z)\) a distance \(z\) above the plane, at target time \( t > 0\), show that the retarded time is positive \((t_r > 0)\) at source points in the \( x-y \) plane for which \( r < \sqrt{c^2t^2 - z^2} \) (where \( r = \sqrt{x^2 + y^2} \)).

(b) Show that the retarded vector potential is

\[
A_x(z,t) = \begin{cases} 
0, & t < z/c \\
(\mu_0 K_0/2) (ct - z), & t > z/c 
\end{cases}
\]

**Hint:** You should encounter an area integral of the form \( \int_0^{\sqrt{c^2t^2 - z^2}} \text{integrand} \ 2\pi r \ dr \), which can be evaluated via the substitution \( u = z^2 + r^2 \).

(c) Also find \( A_y(r,t), A_z(r,t), \) and \( V(r,t) \). (Remember, the sheet is neutral.)

(d) Find \( E(z,t) \) and \( B(z,t) \).
Problem 2 (10 points)

An electric dipole antenna is oriented along the $z$ axis. The maximum electric dipole moment is $p_0$, the angular frequency is $\omega$, and the total power being radiated is $P$.

(a) Starting from the standard expression for the power (no derivation needed), solve for $p_0$ in terms of $P$ and other quantities.

(b) Rewrite the standard formula for the intensity of radiation $I(r, \theta)$ far from the source, expressing your answer in terms of $P$ (i.e., eliminating $\omega$).

(c) The total power being radiated is $P = 10$ kW. What is the intensity of electromagnetic radiation incident on a receiving dish 1 km away (at the same $z$ elevation)?

(d) What is the direction of polarization of the EM wave incident on the receiving dish?

(e) How would your answers to (c) and (d) change, if at all, if it were instead a vertical magnetic dipole antenna ($m \parallel \hat{z}$) radiating the same total power of 10 kW?

\[
I = \frac{\mu_0 p_0^2 \omega^4}{32 \pi^2 c} \frac{\sin^2 \theta}{r^2}
\]

\[
P = \frac{\mu_0 p_0^2 \omega^4}{12 \pi c}
\]

(a) \( p_0 = \left( \frac{12 \pi c P}{\mu_0 \omega^4} \right)^{\frac{1}{2}} \)

(b) \[ I = \frac{\mu_0 \omega^4}{32 \pi^2 c} \frac{12 \pi c}{\mu_0 \omega^4} P \frac{\sin^2 \theta}{r^2} \]

\[ I = \frac{3P}{8 \pi} \frac{\sin^2 \theta}{r^2} \]

(c) \[ I = \frac{3}{8 \pi} \left( \frac{1}{(10^3)^2} \right) = \frac{3}{8 \pi} \times 10^{-2} = 4.19 \text{ mW/m}^2 \]

(d) $\vec{E} \parallel \hat{z}$ (e.g., $\parallel \hat{\phi}$)

(e) Answer to (c) doesn't change, but EM wave has $\vec{B} \parallel \hat{z}$ so is polarized in $\hat{\phi}$ direction.
**Problem 3** (15 points)

In inertial frame $S$, the $x$-$y$ plane is filled by a planar charge density $\sigma$ that is moving in the $+\hat{x}$ direction at speed $v$. The frame in which the charged sheet is at rest is called frame $S'$; that is, frame $S'$ also moves at velocity $+v\hat{x}$ relative to frame $S$. (Note: $\sigma$ is the surface charge as measured in frame $S$, not in frame $S'$.) We are only interested in the $E$ and $B$ fields above the plane ($z > 0$).

(a) Warm-up: the fields in frame $S$ are $E_z = \sigma/2\varepsilon_0$, $B_y = -\mu_0\sigma v/2$, and other components zero. Working entirely in frame $S$, explain briefly why $E_z = \sigma/2\varepsilon_0$, $B_y = -\mu_0\sigma v/2$, and other components are zero.

(b) By transforming the fields, find the components of $E'$ and $B'$ in frame $S'$. Simplify your results by eliminating $\varepsilon_0$ in favor of $\mu_0$ or vice versa. Also note that $\gamma(1 - v^2/c^2) = 1/\gamma$.

(c) Now check your result by doing the problem the other way: find the charge density $\sigma'$ seen by an observer in $S'$, and then compute the fields that you expect from the sources by working entirely in $S'$.