THE STATE UNIVERSITY OF NEW JERSEY

RUTGERS

Department of Physics and Astronomy • 136 Frelinghuysen Road • Piscataway New Jersey 08855-0849
Telephone: (732) 445-4839 • Fax: (732) 445-4991 • e-mail: bart@physics.rutgers.edu

Physics 386, Spring 2008 Prof. Bartynski; Grader: Yi Zhang
Electromagnetism (II) **Due Mon. 25-Feb. 2008**

PROBLEM SET 4

Reading: Griffiths, Chapters 9.1 – 9.2

1. Griffiths 9.1

2. Griffiths 9.9

3. Calculate the Poynting vector for the plane waves described in parts (a) and (b) of Griffiths 9.9.

4. Griffiths 9.33(a) [NOTE: See hint on next page]

5. In class, and in Griffiths, we showed $f(x, t) = g(x - vt)$ solves the wave equation. Show by substitution that $f(x, t) = h(x + vt)$ is also a solution. Show that a function of the form $f(x, t) = h(x + vt)$ represents a wave moving to the left with velocity v.

6. Suppose that the electromagnetic field in a region of space is the superposition of two plane waves of different frequencies (and wave numbers). The electric field is given by:

$$\vec{E}(z,t) = \left[E_o \cos(k_1 z - \omega_1 t) + E_o \cos(k_2 z - \omega_2 t)\right]\hat{x}$$

Show that this electric field solves the wave equation.

(a) Show that this electric field solves the wave equation.
(b) Show, using trig identities, that ***E*** can be written as:

$$\vec{E}(z,t) = 2E_o \cos(\tfrac{\Delta k}{2} z - \tfrac{\Delta\omega}{2} t)\cos(\bar{k}z - \bar{\omega}t)\hat{x}$$

Where $\Delta k = (k_1 - k_2)$; $\Delta\omega = (\omega_1 - \omega_2)$; $\bar{k} = (1/2)(k_1 + k_2)$; and $\bar{\omega} = (1/2)(\omega_1 + \omega_2)$

(c) Repeat part (b) by manipulating the complex waves and then taking the real part: $\vec{E}(z,t) = E_o \Re\{e^{i(k_1 z - \omega_1 t)} + e^{i(k_2 z - \omega_2 t)}\}\hat{x}$

[Note: use the following properties of the complex exponential: $e^{i\alpha} = (e^{i\alpha/2})(e^{i\alpha/2})$ and $1 = (e^{i\alpha/2})(e^{-i\alpha/2})$]

(d) Suppose that $\omega_1 \cong \omega_2$. Show that the electric field can be written as:

$$\vec{E}(z,t) = A(z,t)\cos(kz - \omega t)\hat{x}$$

Where $A(z, t)$ can be thought of as the amplitude that varies slowly in space and time, and $\bar{k} \approx k$; $\bar{\omega} \approx \omega$. This is of course the familiar beating phenomenon when two waves of nearly identical frequency are superposed. Note that for electromagnetic waves, the full effect requires that the electric fields of both waves be parallel.

HINT For question 4 of Problem Set 4 (i.e., Griffiths 9.33(a))

The procedure you should take is the following:

- Verify that the given form of **E** satisfies Gauss's Law.

- Use Faraday's law to determine $\frac{\partial \vec{B}}{\partial t}$ from $\vec{\nabla} \times \vec{E}$. Then integrate with respect to time to find **B**.

- Verify that $\vec{\nabla} \bullet \vec{B} = 0$, confirming Gauss's Law for the magnetic field.

- Calculate $\vec{\nabla} \times \vec{B}$ using the **B** you determined from Faraday's Law and then calculate $\frac{\partial \vec{E}}{\partial t}$ using the form of **E** that is given, and verify that Ampere's Law (with Maxwell's correction of course) is obeyed.

Do not be concerned that there is a component of the **B** field in the $\hat{r}$ direction. This component drops off faster than 1/r and thus does not contribute to the radiation of energy. These rapidly decreasing components are part of the so-called "near field."