Physics 386
Electromagnetism

Spring 2007 - Prof. Bartynski

Final Exam

Monday, 7-May-2007
8:00 AM – 11:00 AM

Closed Book. Closed Notes.
Calculator OK, Three Cheat Sheet OK.

Do not open this exam until instructed to do so.
Please fill out the information on the cover of your blue book.
Answer all 6 problems.

Possibly useful information:

$$\tfrac{1}{T}\int \cos^2\left(\vec{k}\bullet\vec{r}-\omega t\right)=\tfrac{1}{2} \quad ; \quad \int \sin^2\theta d\theta = \tfrac{\theta}{2}-\tfrac{1}{4}\sin(2\theta) \quad ; \quad \int \sin^3(\theta)d\theta = -\tfrac{1}{3}(\cos\theta)(\sin^2\theta+2)$$

$$\int \sin(kr-\omega t)dt = \tfrac{1}{\omega}\cos(kr-\omega t) \quad ; \quad \int \cos(kr-\omega t)dt = -\tfrac{1}{\omega}\sin(kr-\omega t)$$

$$\tfrac{\partial}{\partial r}[\cos(kr-\omega t)] = -k\sin(kr-\omega t) \quad ; \quad \tfrac{\partial}{\partial r}[\sin(kr-\omega t)] = k\cos(kr-\omega t) \quad ; \quad \sin\theta d\theta = -d(\cos\theta)$$

$$\frac{1}{\Re_\pm}=\frac{1}{r}\left(1\pm\frac{d}{2r}\cos\theta\right)=\frac{1}{r}\left(1\pm\frac{A}{r}\right) \quad ; \quad \Re_\pm = r\left(1\mp\frac{d}{2r}\cos\theta\right)=r\left(1\mp\frac{A}{r}\right) \; ;$$

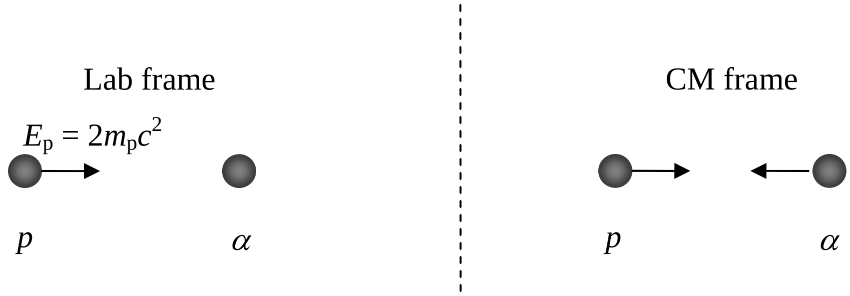
$$\cos[\omega(t-\Re_\pm/c)]=\cos[\omega(t-r/c)]\mp\frac{\omega d}{2c}\cos\theta\sin[\omega(t-r/c)]=(B\mp C)$$

Binomial expansion: $(1+x)^n = 1+nx+\frac{n(n-1)}{2!}x^2+\ldots$ where $x^2<1$

Some useful constants: $e = 1.6 \times 10^{-19}$ C; $\varepsilon_o = 8.85 \times 10^{-12}$ C^2/Nm^2 ; $\mu_o = 4\pi \times 10^{-7}$ N/A^2

30 pts

1) A proton (rest mass = m_p) is moving towards an α-particle [rest mass = $2(m_p + m_n) \approx 4m_p$]. In the laboratory frame, where the α -particle is at rest, the proton has a total energy that is equal to twice its rest energy. What is the velocity of the center of momentum frame of this system with respect to the laboratory frame? [Give your answer in terms of c.]



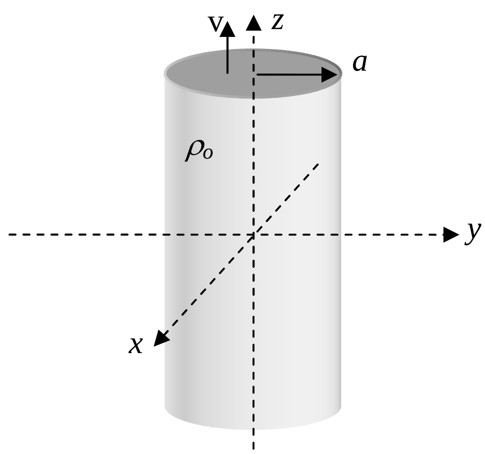
2) In an inertial frame labeled O, the electric and magnetic fields a certain region of space are given by

$$\vec{E} = E_o\left(\frac{\sqrt{3}}{2}\hat{y} + \frac{1}{2}\hat{z}\right) \text{ and } \vec{B} = B_o\left(-\frac{1}{2}\hat{y} + \frac{\sqrt{3}}{2}\hat{z}\right)$$

Consider a second inertial frame, O^*, that is moving with a velocity $\vec{v} = v\hat{x}$ with respect to O.

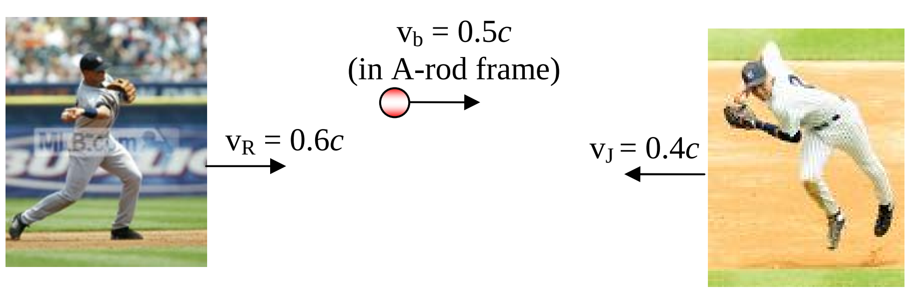
(a) What are the electric and magnetic fields in the O^* frame?
(b) For what speed v (expressed in terms of E_o and B_o) is the electric field in O^* equal to zero?
(c) Is the magnetic field in O^* equal to zero at this speed?
(d) Suppose a point charge q is at rest in O^*, which is moving with respect to O at the speed you determined in part (b). What is the force on the charge? Explain your answer.

3) A long thick cylinder has a radius a. In the xyz-frame, the has a uniform volume charge density ρ and moves at a constant velocity $\vec{v} = v\hat{z}$. Find all of the entries to the electromagnetic field tensor $F^{\mu\upsilon}$ at a point (x, y) in the xy-plane **inside** the cylinder. [You may want to start in cylindrical coordinates and then recast into Cartesian.]



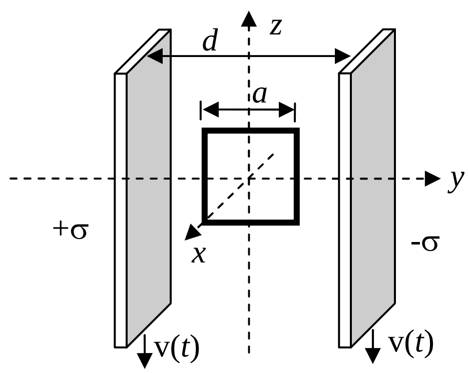
4) In the bottom of the 8th inning at Yankee Stadium, Alex Rodriguez caught a long fly ball deep in left field. Rodriguez starts running towards second with a speed of 0.6c (with respect to the playing field) and throws the ball towards second base with a velocity of 0.5c in his rest frame (after all, he is the $250M man!). The shortstop, Derek Jeter, needs to cut off the throw and runs towards Rodriguez with a speed of 0.4c (with respect to the field).

- (a) What is the velocity of the ball in a frame at rest with respect to the field?
- (b) What is Rodriguez's velocity in a frame at rest with respect to Jeter?
- (c) What is the velocity of the ball in a frame at rest with respect to Jeter?
- (d) Who won the game? (Extra Credit)



5) A clever experimental physicist (is there any other kind???) sets up the following experiment to measure the gravitational constant, g. A large parallel plate capacitor is oriented with the gap parallel to the *z*-axis. The separation between the plates is *d* and there is a surface charge density $+\sigma$ on the left plate and $-\sigma$ on the right plate. Inside the gap a square loop of wire with total resistance R and sides of length *a* lies in the *yz*-plane. The loop is held fixed and the capacitor is released to fall in response to the earth's gravity. [Note: neglect relativistic effects on charge density, retarded times, etc.] Give your answers to the following questions in terms of *g*.

(a) What is the velocity of the plates, v(*t*), as a function of time?
(b) What is the (time dependent) surface **current** density of each plate?
(c) What is the magnetic field (magnitude and direction) between the plates?
(d) In what direction is the induced current in the loop? [Show a clear sketch!]
(e) What is the magnitude of the induced current in the loop?
(f) What is the force exerted on the part of the loop at $z = a/2$?
(g) What is the net force on the loop?



6) A large circular parallel plate capacitor, with plates of radius *R* separated by a distance *d*, is connected to wires that carry a current *I* as shown in the figure. Assume that $\sigma(t)$, the charge density on the surface of the plates, is uniform and that the nature of the fields near the edge of the plates is the same as that well inside the plates.

(a) What is the (time dependent) electric field between the plates? [Give both magnitude and direction.]
(b) What is the magnetic field [magnitude and direction] between the plates?
(c) What is the Poynting vector at the edge of the gap ($r = R$)?
(d) Integrate the Poynting vector over the cylindrical surface that bounds the gap and show that this is equal to the rate at which energy is being stored in the capacitor [recall that for a capacitor $W = \frac{Q^2}{2C}$ where here $C = \frac{\varepsilon_o A}{d}$ (A = area of plate)].