Physics 386
Electromagnetism

Spring 2007 - Prof. Bartynski

Exam II- Redux

TAKE HOME EXAM
OPEN BOOK, OPEN NOTES
WORK ALONE!!!

Do all four (4) problems.

DUE: Monday, 16-April-2007
4:30 PM (my office or to my Ass't. Nancy in NPL 210)

30 pts 1) For an electromagnetic wave in a conductor, the (real) electric and magnetic fields have the form:

$$\boldsymbol{E}(z,t) = E_o e^{-\kappa z}\cos(kz - \omega t + \delta_E)\hat{\boldsymbol{x}}\,; \quad \boldsymbol{B}(z,t) = B_o e^{-\kappa z}\cos(kz - \omega t + \delta_E + \phi)\hat{\boldsymbol{y}}$$

$$\text{where } \frac{B_o}{E_o} = \sqrt{\mu\varepsilon\sqrt{1+\left(\frac{\sigma}{\varepsilon\omega}\right)^2}}$$

a) In what direction does this wave propagate?
b) Using the parameters in the equations for $\boldsymbol{E}$ and $\boldsymbol{B}$, what is the speed of propagation of the wave?
c) What is the time averaged energy density contained in this wave? At what position is the time averaged energy density reduced by a factor of e^{-1} from what it is at $z = 0$?
d) We say in an expression like this that "B lags behind E by ϕ." Explain why the phrase "lags" is used.

20 pts 2) In class (and in Griffiths) we came up with the following expression for the electric field associated with a moving point charge:

$$\vec{\boldsymbol{E}}(\vec{r},t) = \frac{q}{4\pi\varepsilon_o}\frac{\Re}{(\vec{\Re}\bullet\vec{\boldsymbol{u}})^3}[(c^2 - \mathrm{v}^2)\vec{\boldsymbol{u}} + \vec{\Re}\times(\vec{\boldsymbol{u}}\times\vec{\boldsymbol{a}})]$$

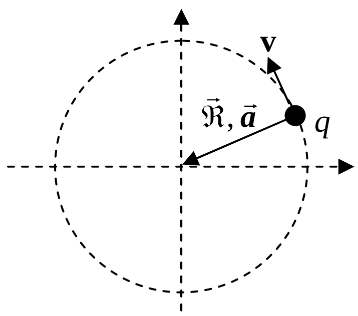
where $\vec{\boldsymbol{u}} = c\hat{\Re} - \vec{\mathbf{v}}$, $\vec{\mathbf{v}}$ is the velocity, $\vec{\boldsymbol{a}}$ is the acceleration, and $\vec{\Re} = \vec{\boldsymbol{r}} - \vec{\boldsymbol{r}}\,'$.
Consider a charge q that is executing uniform circular motion on a circle of radius R. The directions of vectors $\vec{\Re}$, $\vec{\boldsymbol{a}}$, and $\vec{\mathbf{v}}$ are shown in the diagram and are given by

$$\vec{\Re} = -R\hat{\boldsymbol{r}}, \text{ and } \vec{\boldsymbol{a}} = -a\hat{\boldsymbol{r}} = -\tfrac{\mathrm{v}^2}{\mathrm{R}}\hat{\boldsymbol{r}}\ ; \quad \vec{\mathbf{v}} = \mathrm{v}\hat{\phi}\,.$$

(a) Show that the electric field is given by: $\vec{\boldsymbol{E}}(\vec{r},t) = \dfrac{q(c\vec{\boldsymbol{u}} - \Re\vec{\boldsymbol{a}})}{4\pi\varepsilon_o(\mathrm{c}\Re)^2}$.

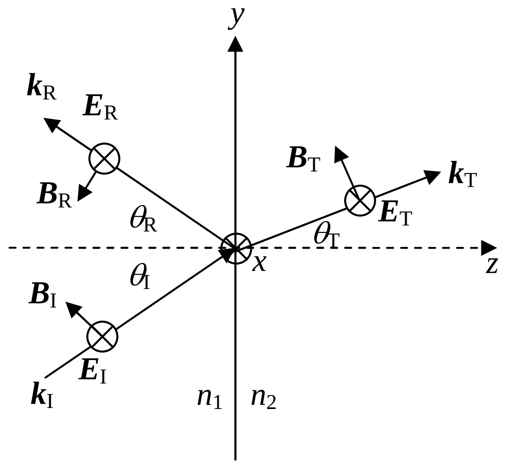
(b) Rewrite this expression in terms of $\hat{\boldsymbol{r}}$ and $\hat{\phi}$.

(c) What is the direction of $\boldsymbol{E}$ in the limit that $\mathrm{v} \to c$?



30 pts

3) Consider a plane polarized electromagnetic wave incident on an interface between dielectric medium 1 ($z < 0$) and medium 2 ($z > 0$) where the electric field is polarized perpendicular to the plane of incidence as shown in the diagram below. The electric field points in the $+x$ direction (into the page), while the magnetic field has components in the zy-plane.



a) Find an expression for the ratio of the amplitudes $\frac{E_{0R}}{E_{0I}}$ and $\frac{E_{0T}}{E_{0I}}$ in terms of n_1 and n_2, θ_I, and θ_T.

b) Find an expression for the reflection coefficient, R, and the transmission coefficient, T.

c) Show that R + T = 1.

20 pts

4) Consider the loop shown in the figure below. A current $I(t) = Kt$ flows in the loop in the sense shown.

a) What is the vector potential, $\boldsymbol{A}(\boldsymbol{r}, t)$, at the origin?

b) What is the electric field at the origin?

c) From what you've calculated here, can you determine $\boldsymbol{B}$ at the origin? Why?

[Hint: recall that $\oint d\vec{l} = 0$ and that $\int_{\vec{a}}^{\vec{b}} d\vec{l} = \vec{b} - \vec{a}$ that is, the vector from $\vec{a}$ to $\vec{b}$.]