Physics 386
Electromagnetism

Spring 2007 - Prof. Bartynski

Exam I

Friday, 23-February-2007
1:40 PM – 3:00 PM

Closed Book. Closed Notes.
Calculator OK, One Cheat Sheet OK.

Do not open this exam until instructed to do so.
Please fill out the information on the cover of your blue book.
Answer all 4 problems.

Possibly useful information:

$$\tfrac{1}{T}\int \cos^2\left(\vec{k}\bullet\vec{r}-\omega t\right)=\tfrac{1}{2} \quad ; \quad \int \sin^2\theta d\theta = \tfrac{\theta}{2}-\tfrac{1}{4}\sin(2\theta) \quad ; \quad \int \sin^3(\theta)d\theta = -\tfrac{1}{3}(\cos\theta)(\sin^2\theta+2)$$

$$\int \sin(kr-\omega t)dt = \tfrac{1}{\omega}\cos(kr-\omega t) \quad ; \quad \int \cos(kr-\omega t)dt = -\tfrac{1}{\omega}\sin(kr-\omega t)$$

$$\tfrac{\partial}{\partial r}[\cos(kr-\omega t)] = -k\sin(kr-\omega t) \quad ; \quad \tfrac{\partial}{\partial r}[\sin(kr-\omega t)] = k\cos(kr-\omega t) \quad ; \quad \sin\theta d\theta = -d(\cos\theta)$$

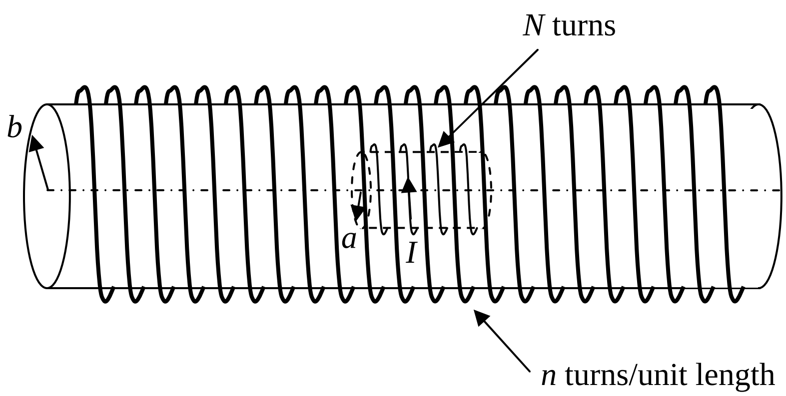
Binomial expansion: $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots$ where $x^2 < 1$

Some useful constants: $e = 1.6 \times 10^{-19}$ C; $\varepsilon_o = 8.85 \times 10^{-12}$ C^2/Nm^2 ; $\mu_o = 4\pi \times 10^{-7}$ N/A^2

20 pts 1) A LONG solenoid of total resistance R has a circular cross section of radius b and n turns per unit length. Coaxial with this long solenoid is a second SHORT solenoid of radius a and a total of N turns (wound in the same orientation as the long solenoid) carrying current I_o for $t < 0$.

(a) Suppose a switch is opened at time $t = 0$ so that for $t > 0$ the current in the SHORT solenoid goes as $I = I_o e^{-t/\tau}$. What is the magnitude of the induced current in the LONG solenoid? [Hint: recall $M_{21} = M_{12}$]

(b) Indicate with a sketch the direction of the induced current in the LONG solenoid.



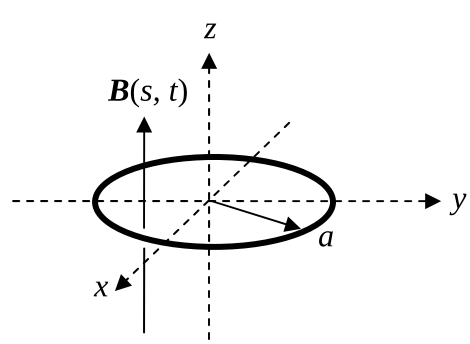
30 pts 2) A rigid conducting wire is formed into a circular loop of resistance R and radius a centered at the origin and lying in xy-plane. This region of space is filled with a time-varying magnetic field oriented in the z-direction and depends upon distance from the z-axis as $\vec{B}(s,t) = Ast\hat{z}$, where A is a positive constant.

(a) What is the magnitude of the current induced in the loop?

(b) In what direction does the induced current flow (use a diagram)?

(c) In what direction is the force exerted on a small segment of the loop, $d\mathbf{l}$, which is centered at some arbitrary point on the loop?

(d) What is the total force the loop must withstand to prevent changing shape?

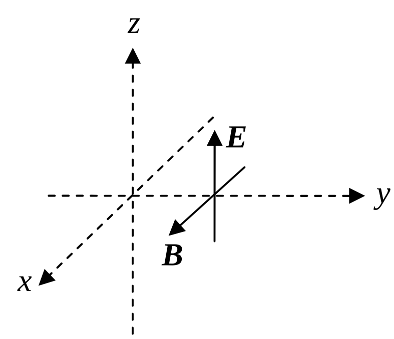


20 pts 3) A region of space that contains the *xy*-plane is filled with a uniform electric field oriented in the *z*-direction ($\vec{E} = E_o\hat{z}$), and a uniform magnetic oriented in the *x*-direction ($\vec{B} = B_o\hat{x}$).

(a) Find all components of the Maxwell Stress Tensor in this region.

(b) Recall that for static fields (*i.e.* a time independent Poynting vector: $\frac{d\vec{S}}{dt} = 0$) the force on all charges in a volume is given by $\vec{F} = \oint_S \overleftrightarrow{T} \bullet d\vec{a}$ where $\mathcal{S}$ is the surface that bounds the volume $\mathcal{V}$. Find the force per unit area, that is, find $\overleftrightarrow{T} \bullet d\vec{a}$, on the *xy*-plane contributing to the total force on the charges in the half space $z < 0$.

(c) What should be the value of B_o in terms of E_o so that the charges in $z < 0$ experience no force?



30 pts 4) In class we discussed the circularly polarized plane electromagnetic wave. The electric field is given by

$$\vec{E} = E_o \cos(kz - \omega t)\hat{x} + E_o \sin(kz - \omega t)\hat{y}\ .$$

(a) What is the *k*-vector, ***k***, for this wave?

(b) Use Faraday's law $\left(\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}\right)$ to determine the magnetic field of this EM wave.

(c) Verify that ***B*** you found in part (b) can be written as $\vec{B} = \frac{1}{c}\left(\hat{k} \times \vec{E}\right)$.

(d) Sketch the orientations of ***E*** and ***B*** at $z = 0$ for times $t = 0$, $(\pi/2\omega)$, (π/ω), $(3\pi/2\omega)$, and $(2\pi/\omega)$. [Please make 5 separate sketches and clearly label the time and ***E*** and ***B***.]