Physics 386
Electromagnetism

Spring 2006 - Prof. Bartynski

Final Exam

Monday, 8-May-2006
8:00 AM – 11:00 AM

Closed Book. Closed Notes.
Calculator OK, Three Cheat Sheet OK.

Do not open this exam until instructed to do so.
Please fill out the information on the cover of your blue book.
Answer all 6 problems.

Possibly useful information:

$$\tfrac{1}{T}\int \cos^2\left(\vec{k}\bullet\vec{r}-\omega t\right)=\tfrac{1}{2} \quad ; \quad \int \sin^2\theta d\theta = \tfrac{\theta}{2}-\tfrac{1}{4}\sin(2\theta) \quad ; \quad \int \sin^3(\theta)d\theta = -\tfrac{1}{3}(\cos\theta)(\sin^2\theta+2)$$

$$\int \sin(kr-\omega t)dt = \tfrac{1}{\omega}\cos(kr-\omega t) \quad ; \quad \int \cos(kr-\omega t)dt = -\tfrac{1}{\omega}\sin(kr-\omega t)$$

$$\tfrac{\partial}{\partial r}[\cos(kr-\omega t)] = -k\sin(kr-\omega t) \quad ; \quad \tfrac{\partial}{\partial r}[\sin(kr-\omega t)] = k\cos(kr-\omega t) \quad ; \quad \sin\theta d\theta = -d(\cos\theta)$$

$$\frac{1}{\Re_\pm} = \frac{1}{r}\left(1 \pm \frac{d}{2r}\cos\theta\right) = \frac{1}{r}\left(1 \pm \frac{A}{r}\right) \quad ; \quad \Re_\pm = r\left(1 \mp \frac{d}{2r}\cos\theta\right) = r\left(1 \mp \frac{A}{r}\right) \ ;$$

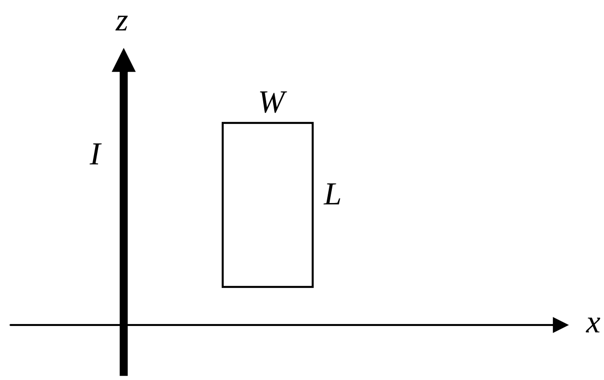
$$\cos[\omega(t-\Re_\pm/c)] = \cos[\omega(t-r/c)] \mp \frac{\omega d}{2c}\cos\theta\sin[\omega(t-r/c)] = (B \mp C)$$

Binomial expansion: $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots$ where $x^2 < 1$

Some useful constants: $e = 1.6 \times 10^{-19}$ C; $\varepsilon_o = 8.85 \times 10^{-12}$ C^2/Nm^2 ; $\mu_o = 4\pi \times 10^{-7}$ N/A^2

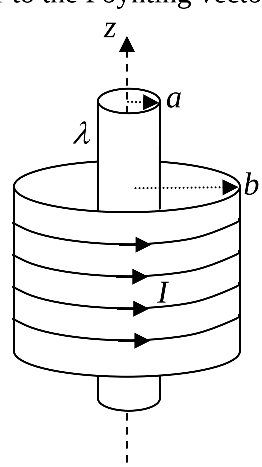
20 pts 1) A rectangular loop of wire of length L, width W, and resistance R lies in the *zx*-plane. A separate long straight wire lies along the *z*-axis and carries a current I in the +*z*-direction as shown in the diagram below. Assume the loop moves in the +*x*-direction at a constant velocity *v*.

(a) What is the direction of the current induced in the wire?
(b) What is the magnitude of the current induced in the wire?
(c) What is the magnitude and direction of the force, if any, that must be applied to keep the loop moving with constant velocity?
(d) Repeat parts (a), (b) and (c) assuming that, instead, the loop moves with a constant velocity parallel to the *z*-axis.



20 pts 2) A long thin conducting cylinder of radius *a* and uniform charge per unit length λ lies along the *z*-axis. The cylinder is surrounded by a long coaxial solenoid of radius *b*, with *n* turns per unit length and carrying current *I* per turn in the direction shown in the diagram.

(a) What is the **E** in the regions $0 < s < a$; $a < s < b$, $b < s$?
(b) What is the **B** in those three regions?
(c) What is the Poynting vector **S** and where is it nonzero?
(d) Determine power per unit length carried by these (static!) fields across a surface perpendicular to the Poynting vector.



30 pts 3) The scalar and vector potentials of an oscillating electric dipole are given by:

$$V(r,\theta,t) = \frac{p_o \cos\theta}{4\pi\varepsilon_o r}\left\{-\frac{\omega}{c}\sin[\omega(t-r/c)] + \frac{1}{r}\cos[\omega(t-r/c)]\right\}$$

$$\boldsymbol{A}(r,\theta,t) = -\frac{\mu_o p_o \omega}{4\pi r}\sin[\omega(t-r/c)](\cos\theta\hat{\boldsymbol{r}} - \sin\theta\hat{\boldsymbol{\theta}})$$

Show that these potentials satisfy the Lorentz condition: $\nabla \bullet \boldsymbol{A} = -\mu_o\varepsilon_o \frac{\partial V}{\partial t}$.

30 pts 4) Two events, A and B, are represented by the 4-vectors:

$$x_A^\mu = [x^0, x^1, x^2, x^3] = [5, 2, 0, 0] \text{ and } x_B^\mu = [9, 1, 0, 0]$$

(a) Is the invariant interval separating these events space-like or time-like? If it is time-like, find the velocity of the frame in which these events occur at the same spatial point. If it is space-like, find the velocity of the frame where these two events occur at the same instant in time.

(b) Repeat (a) for the two events C and D given by:

$$x_C^\mu = [3, 14, 0, 0] \text{ and } x_D^\mu = [7, 2, 0, 0]$$

30 pts 5) A point charge q moves along the x-axis at a constant velocity v through a region with a uniform electric field and magnetic field given by $\boldsymbol{E} = E_o\hat{\boldsymbol{z}}$ and $\boldsymbol{B} = -B_o\hat{\boldsymbol{y}}$ and experiences no force.

What are the electric and magnetic fields, $\boldsymbol{E}^*$ and $\boldsymbol{B}^*$, in the rest frame of the charge? Does the charge experience a force in this frame? Why or why not?

30 pts 6) A long uniform line of charge with λ charge per unit length moves along the $+z$-axis with a velocity v.

Fill in all the elements of the electromagnetic field tensor $F^{\mu\upsilon}$ for an arbitrary point $(x, y, 0)$ in the xy-plane (Be sure that the only variables in your answers are x, y, and z, that is, do not use s or θ).