Physics 386
Electromagnetism

Spring 2006 - Prof. Bartynski

Exam II

Friday, 7-April-2006
1:40 PM – 3:00 PM

Closed Book. Closed Notes.
Calculator OK, Two Cheat Sheet OK.

Do not open this exam until instructed to do so.
Please fill out the information on the cover of your blue book.
Answer all 4 problems.

Possibly useful information:

$$\tfrac{1}{T}\int \cos^2\left(\vec{k}\bullet\vec{r}-\omega t\right)=\tfrac{1}{2} \quad ; \quad \int \sin^2\theta d\theta = \tfrac{\theta}{2}-\tfrac{1}{4}\sin(2\theta) \quad ; \quad \int \sin^3(\theta)d\theta = -\tfrac{1}{3}(\cos\theta)(\sin^2\theta+2)$$

$$\int \sin(kr-\omega t)dt = \tfrac{1}{\omega}\cos(kr-\omega t) \quad ; \quad \int \cos(kr-\omega t)dt = -\tfrac{1}{\omega}\sin(kr-\omega t)$$

$$\tfrac{\partial}{\partial r}[\cos(kr-\omega t)] = -k\sin(kr-\omega t) \quad ; \quad \tfrac{\partial}{\partial r}[\sin(kr-\omega t)] = k\cos(kr-\omega t) \quad ; \quad \sin\theta d\theta = -d(\cos\theta)$$

$$\frac{1}{\Re_\pm} = \frac{1}{r}\left(1 \pm \frac{d}{2r}\cos\theta\right) = \frac{1}{r}\left(1\pm\frac{A}{r}\right) \quad ; \quad \Re_\pm = r\left(1 \mp \frac{d}{2r}\cos\theta\right) = r\left(1\mp\frac{A}{r}\right) \ ;$$

$$\cos\left[\omega\left(t-\Re_\pm/c\right)\right] = \cos\left[\omega\left(t-r/c\right)\right] \mp \frac{\omega d}{2c}\cos\theta\sin\left[\omega\left(t-r/c\right)\right] = \left(B \mp C\right)$$

Binomial expansion: $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots$ where $x^2 < 1$

Some useful constants: $e = 1.6 \times 10^{-19}$ C; $\varepsilon_o = 8.85 \times 10^{-12}$ C^2/Nm^2 ; $\mu_o = 4\pi \times 10^{-7}$ N/A^2

20 pts 1) The Larmor formula states that the power radiated by a point charge is:

$$P = \frac{\mu_o q^2 a^2}{6\pi c}$$

where a is the particle's acceleration. Suppose that the position of a charge q, which is at x = 0 for t < 0, goes as $x(t) = x_o\left(1 - e^{-t/\tau}\right)$ for t > 0. **What** is the total energy radiated by the particle while it is not at rest?

30 pts 2) Suppose the xy-plane divides space in to the regions: z < 0 which is vacuum, and z > 0 which is filled with a metal. An EM wave propagating in the +z direction with electric field given by $E_x = E_{ox} e^{i(kz-\omega t)}$ is normally incident on the interface. The electrons in the metal respond to the local applied electric field as free charges, so $F = ma$ becomes $m\ddot{x} = -eE_{ox}e^{-i\omega t}$. [Note: you don't need to worry about σ in this problem. Treat the metal as a dielectric.]

(a) **Show that** the (complex, time dependent) dipole moment, $p(t) = -ex(t)$, for such an electron is $p(t) = -\frac{e^2 E_{ox}}{m\omega^2} e^{-i\omega t} = -\frac{e^2}{m\omega^2} E_x$ so that the (complex) polarization $P(t)$ is given by $P(t) = -\frac{Ne^2}{m\omega^2} E_x$ where N is the number of electrons per unit volume.

(b) From the result of (a), **show that** the dielectric constant of the metal can be written as

$$\varepsilon = \varepsilon_o\left(1 - \frac{Ne^2}{m\varepsilon_o\omega^2}\right).$$

(c) Recall that in linear media the wave equation for the electric field in matter is given by:

$$\frac{\partial^2 E}{\partial x^2} = \mu_o \varepsilon \frac{\partial^2 E}{\partial t^2} \qquad \text{(where I have assumed } \mu = \mu_o\text{)}.$$

Show that the electric field can be written as $E_x = E_{ox} e^{i(nk_o z - \omega t)}$ where k_o is the wave number for this wave in vacuum. **Write** an expression for n (which is the index of refraction).

(d) **Show that**, for a frequencies below some cutoff, ω_c, the electric field in the metal goes as: $E_x = E_{ox} e^{-\kappa z} e^{-i\omega t}$ so that the wave does not propagate in the metal, but rather its amplitude decays exponentially as the wave propagates deeper into the metal. **Find** an expression for is κ. **Find** an expression for the cutoff frequency ω_c.

[So, in this simple model, for $\omega < \omega_c$, the metal is perfectly reflecting, Above ω_c, it is perfectly transmitting. This phenomenon, which is prevalent in simple metals like sodium, is known as the ultraviolet transparency of metals.]

20 pts 3) The scalar and vector potentials in some region of space away from the origin are given by:

$$V = -\frac{K^2}{r} \text{ and } \vec{A} = A_o \sin(kx - \omega t)\hat{y}$$

(a) **Find *E*** and ***B*** corresponding to these potentials.
(b) **What** physical situation might give rise to such fields?

30 pts 4) When we examined electromagnetic radiation from a point dipole we found that the electric field was given by

$$\vec{E} = -\frac{\mu_o p_o \omega^2}{4\pi}\left(\frac{\sin\theta}{r}\right)\cos[\omega(t - \tfrac{r}{c})]\hat{\theta}$$

.

Suppose we consider a quadrupole formed by two dipoles separated by a distance *d*, as illustrated in the figure below.

(a) **Show** using superposition that the electric field at point P, that is well into the radiation zone, is given by:

$$\vec{E} = \frac{\mu_o p_o \omega^3 d}{4\pi c}\left(\frac{\sin\theta\cos\theta}{r}\right)\sin[\omega(t - \tfrac{r}{c})]\hat{\theta}$$

[I suggest you look at the first page of the exam for some useful approximations. In particular, use the “A, B, C” forms to sort out what survives in the radiation zone, and then express the result in terms of the parameters of the problem.]

(b) **What** is the magnetic field at point P?
(c) **What** is the Poynting vector at P?
(d) Recall that we discussed in class that the sky is blue because molecules in the air that are excited by sunlight in turn radiate as isolated dipoles and the frequency dependence of such radiation favored shorter wavelengths. If in stead the molecules radiated like these quadrupoles, **would** the color of the sky be redder, greener, or more violet? **Why**?