Physics 386
Electromagnetism

Spring 2006 - Prof. Bartynski

Exam I

Friday, 24-February-2006
1:40 PM – 3:00 PM

Closed Book. Closed Notes.
Calculator OK, One Cheat Sheet OK.

Do not open this exam until instructed to do so.
Please fill out the information on the cover of your blue book.
Answer all 4 problems.

Possibly useful information:

$$\tfrac{1}{T}\int \cos^2\left(\vec{k}\bullet\vec{r}-\omega t\right)=\tfrac{1}{2} \quad ; \quad \int \sin^2\theta d\theta = \tfrac{\theta}{2}-\tfrac{1}{4}\sin(2\theta) \quad ; \quad \int \sin^3(\theta)d\theta = -\tfrac{1}{3}(\cos\theta)(\sin^2\theta+2)$$

$$\int \sin(kr-\omega t)dt = \tfrac{1}{\omega}\cos(kr-\omega t) \quad ; \quad \int \cos(kr-\omega t)dt = -\tfrac{1}{\omega}\sin(kr-\omega t)$$

$$\tfrac{\partial}{\partial r}[\cos(kr-\omega t)] = -k\sin(kr-\omega t) \quad ; \quad \tfrac{\partial}{\partial r}[\sin(kr-\omega t)] = k\cos(kr-\omega t) \quad ; \quad \sin\theta d\theta = -d(\cos\theta)$$

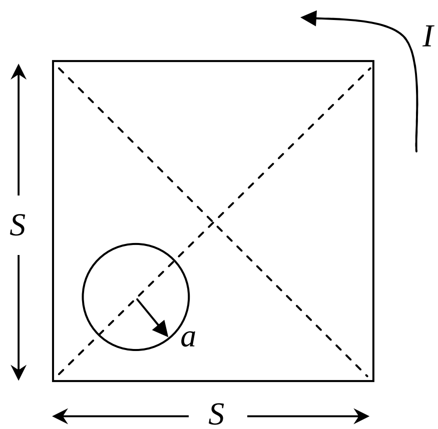
Binomial expansion: $(1+x)^n = 1+nx+\frac{n(n-1)}{2!}x^2+\ldots$ where $x^2 < 1$

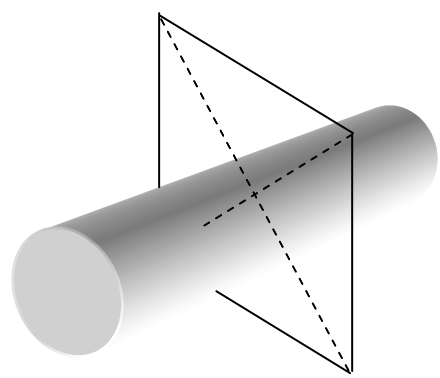
Some useful constants: $e = 1.6 \times 10^{-19}$ C; $\varepsilon_o = 8.85 \times 10^{-12}$ C^2/Nm^2 ; $\mu_o = 4\pi \times 10^{-7}$ N/A^2

20 pts 1) A long solenoid of total resistance R with a circular cross section of radius *a* and *n* turns per meter passes through a square loop of side *S*. The axis of the solenoid is perpendicular to the square loop, and intersects the loop mid way between the center and one of the corners as shown in the diagram.

(a) Suppose a switch is closed at time $t = 0$ so that for $t > 0$ the current in the square loop goes as $I = I_F(1 - e^{-t/\tau})$. What is the magnitude of the induced current in the solenoid? [Hint: recall $M_{21} = M_{12}$]

(b) Indicate with a sketch the direction in which the induced current in the solenoid flows.



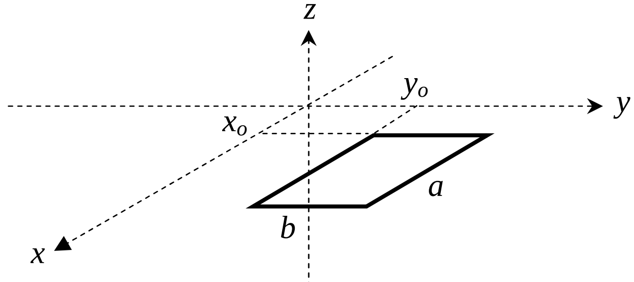


20 pts 2) A rectangle of wire of length *a* in the *x*-direction, length *b* in the *y*-direction, and resistance *R* lies in the *xy*-plane. In this region of space, there is a magnetic field given by $\vec{B} = \beta\left(y^2\hat{x} + x^2\hat{y} + 2xy\hat{z}\right)$. The corner of the loop lies at the point (x_o, y_o).

(a) Find the flux of magnetic field through the loop.

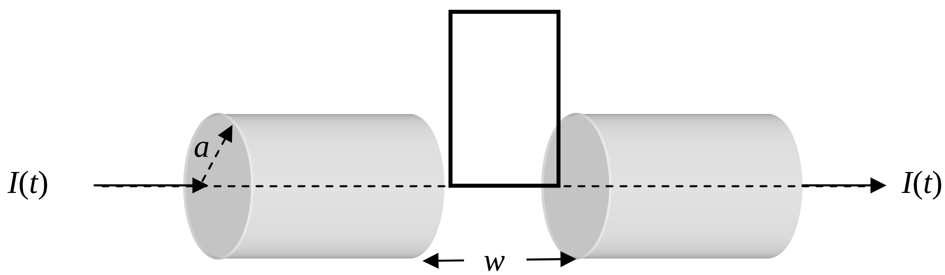
(b) Suppose the loop has a resistance, *R*, and moves in the *x*-direction with a constant velocity $\frac{dx_o}{dt} = v_o$. What is the magnitude of the current induced in the loop?

(c) In what direction does the induced current flow. (You may use a diagram but be very clear about the orientation of your axes). (Assume $v_o > 0$ and motion is restricted to the quadrant with $x_o > 0$ and $y_o > 0$.)



30 pts 3) A long, straight, thick wire of radius *a* has a small gap of width *w*, such that the ends of the wire form a parallel plate capacitor with circular plates. A current I(t) = αt flows in the wire towards the gap from the left and the same current flows away from the gap towards the right as shown. A rectangular loop of wire that lies in a plane containing the axis of the thick wire is located in the gap as shown. Assume all fields are confined to the gap (i.e., ignore fringe fields).

(a) What is the surface charge density $+\sigma$ and $-\sigma$, that accumulates on the left and right "plates" of the capacitor as a function of time?
(b) What is the electric field in the gap as a function of time?
(c) What is the magnetic field in the gap?
(d) What is the magnetic flux through the rectangular loop?
(e) What is the magnitude of the current induced in the loop?
(f) In what direction does the induced current in the loop flow?



30 pts 4) The electric field of a spherical electromagnetic wave (that is, the surfaces of constant phase are spheres) is given (in spherical coordinates) by

$$\vec{E} = \frac{E_o \sin\theta}{r} \cos(kr - \omega t)\hat{\theta} .$$

(a) What is the *k*-vector, ***k***, for this wave?
(b) Use Faraday's law $\left(\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \right)$ to determine the magnetic field of this EM wave [remember to use $\vec{\nabla}$ in spherical coordinates]. (Note only one term should survive.)
(c) Verify that ***B*** you found in part (b) can be written $\vec{B} = \frac{1}{c}\left(\hat{k} \times \vec{E}\right)$.
(d) Find an expression for the Poynting vector $\vec{S} = \frac{1}{\mu_o}\left(\vec{E} \times \vec{B}\right)$. What is the time average of the Poynting vector $\left\langle \vec{S} \right\rangle_t = \frac{1}{T}\int_t^{t+T} \vec{S}dt$
(e) Perform the integral $\oint \left\langle \vec{S} \right\rangle_t \bullet d\vec{a}$ on the surface of a sphere of radius *R* centered at the origin. This represents the total power carried out of the spherical volume by the electromagnetic wave.
(f) How does the power radiated out of the sphere depend on the radius of the sphere?