

Name: _____

Ground rules:

- Open book, open notes, open everything
- A calculator is allowed but will not be useful
- Write your answer directly on these sheets (continue onto back, if necessary)

There are four questions of 25 points each. Pace yourself accordingly.

If you know the formulas for “standard cases” (e.g., the magnetic field around an infinite straight current-carrying wire), you may use these results without derivation unless the problem specifically asks you to derive it.

Partial credit will be given. Do as many parts of a problem as possible. If you are stuck on the first part of a problem, you should still try to say something about the later parts if possible.

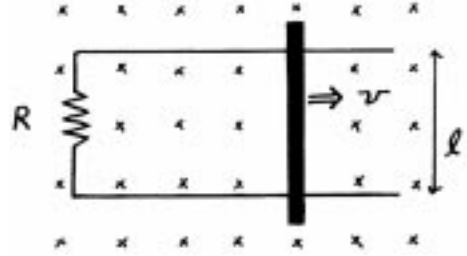
Be sure to check that you have done all parts of all questions.

Feel free to raise your hand to ask a question.

Good luck!!

Problem 1 (25 points)

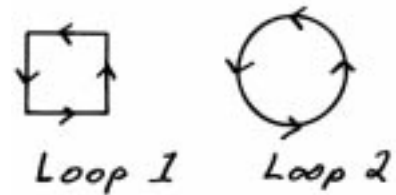
A metal bar slides frictionlessly on two parallel conducting rails a distance l apart as shown, with a resistor R connecting the two rails. A constant and uniform magnetic field B points into the page everywhere.



- If the bar slides to the right at speed v , what current flows through the resistor? In what direction?
- What is the magnetic force on the bar, and in what direction?
- What is the power being dissipated in the resistor?
- Discuss conservation of energy.

Problem 2 (25 points)

Two wire loops lie next to each other as shown. The only things you know are that the resistance of the circular loop is R_2 ; the mutual inductance of the two loops is M ; and the current in the square loop is some given $I_1(t)$.



a) Suppose

$$I_1(t) = I_0, \quad t < 0$$
$$I_1(t) = I_0 \left(1 - \frac{t^2}{\tau^2}\right), \quad 0 < t < \tau$$
$$I_1(t) = 0, \quad t > \tau$$

(For example, perhaps the square loop had been connected to a battery until time $t = 0$, when some switch was opened over a time interval τ .) Find the current $I_2(t)$ induced in the circular loop.

b) Find the total charge Q that flows in the circular loop during this time.

c) Suppose all you know is that $I_1(t)$ falls monotonically from I_0 to 0 during some time. Is the answer to part (b) still the same? If so, show why.

Problem 3 (25 points)

Suppose the electric field is known to be

$$\mathbf{E}(\mathbf{r}, t) = \frac{\alpha}{s} \sin(\omega t) \hat{\mathbf{z}}$$

independent of z and ϕ , where α is a constant, and s is the radial coordinate of the (s, ϕ, z) cylindrical coordinate system. No actual currents flow; that is, $\mathbf{J} = 0$.

a) Find the displacement current $\mathbf{J}_d(\mathbf{r}, t)$.

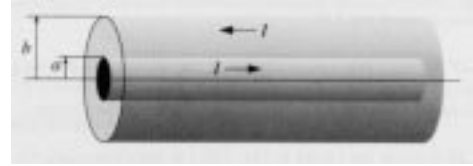
b) Find the magnetic field induced by the displacement current at time $t = 0$.

Hint: Argue that $\mathbf{B} = B(s)\hat{\phi}$ (why?) and compute $B(s)$.

Problem 4 (25 points)

This is a purely *qualitative* problem; no quantitative answers are needed.

Two concentric cylindrical metal shells form a kind of coaxial cable as shown. The same current I flows to the right on the inside shell, and to the left on the outside shell.



- Where is the \mathbf{B} -field zero, and where is it non-zero? In the region where it is non-zero, in which direction does it point?
- Now suppose I is time-dependent, $I(t) = I_0 e^{-t/\tau}$. Where is the \mathbf{E} -field zero, and where is it non-zero? In the regions where it is non-zero, in which direction does it point?
- Where is the Poynting vector non-zero, and in which direction does it point?

For extra credit, see how far you can get with a quantitative solution.