1) a) Charges are given; pattern of \( V \)'s comes from symmetry.

b) When \( S_1 \) and \( S_3 \) are connected, a charge \( \frac{1}{2}Q_0 \) flows from \( S_1 \) to \( S_3 \), so that \( Q_1 = Q_3 = \frac{1}{2}Q_0 \), \( Q_2 = Q_4 = 0 \).

   Why? Because the charges on \( S_2 \) and \( S_4 \) are the same, so the problem has top-bottom mirror symmetry, and the solution should therefore obey \( Q_1 = Q_3 \).

   (But wait: Does it matter that the charges that start on \( S_1 \) and \( S_3 \) are different? Does that ruin the symmetry? No, that would violate uniqueness of the solution. For suppose you started with \( Q_1 = 0 \) and \( Q_3 = Q_0 \); then if you are right, this would lead to a different solution with the final values of \( Q_1 \) and \( Q_3 \) interchanged. That would violate uniqueness. To put it another way: we can specify the problem as a boundary-value problem: find a solution for the case \( V_1 = V_3 \), \( Q_1 + Q_3 = Q_0 \), and \( Q_2 = Q_4 = 0 \). Stated this way, there is no reference to the starting condition, so the solution should have top-bottom symmetry; and that must be the only solution.)

c) After \( S_2 \) and \( S_4 \) are connected, the problem becomes: find a solution for the case \( V_2 = V_3 \), \( Q_1 = \frac{1}{2}Q_0 \), \( Q_2 + Q_3 = \frac{1}{2}Q_0 \), \( Q_4 = 0 \). This problem has mirror symmetry about the diagonal line that runs from upper-left to lower-right, and the result must therefore obey \( Q_2 = Q_3 \) and so they both equal \( \frac{1}{4}Q_0 \). Now \( V_2 = V_3 \) but we can't say much about \( V_1 \) or \( V_4 \).

d) After \( S_1 \) and \( S_4 \) are connected, the problem has symmetry about the other diagonal, and the result is \( Q_1 = Q_2 = Q_3 = Q_4 = \frac{1}{4}Q_0 \) and \( V_1 = V_2 = V_3 = V_4 \).

   Note that other sequences of connections do not necessarily end up with this neat symmetric situation. For example, if you connect 1–3 then 3–4 then 4–2, you end up with a mess!

2) \[
\nabla^2 V = \frac{1}{s} \frac{d}{ds} \left( s \frac{dV}{ds} \right) = 0 \Rightarrow s \frac{dV}{ds} = c \Rightarrow \frac{dV}{ds} = \frac{c}{s} \Rightarrow V = c \ln s + k.
\]

3) \[
V(x, 0, 0) = \frac{\rho}{4\pi\varepsilon_0} \left[ \frac{4}{\sqrt{(x-1)^2 + 1^2 + 1^2}} + \frac{4}{\sqrt{(x+1)^2 + 1^2 + 1^2}} \right]
= \frac{\rho}{\varepsilon_0} \left[ \frac{1}{\sqrt{x^2 - 2x + 1}} + \frac{1}{\sqrt{x^2 + 2x + 1}} \right]
\]

Using a calculator, we get

<table>
<thead>
<tr>
<th>( x )</th>
<th>\text{Quantity in Brackets}</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>1.1547</td>
</tr>
<tr>
<td>0.2</td>
<td>1.1546</td>
</tr>
<tr>
<td>0.4</td>
<td>1.1535</td>
</tr>
<tr>
<td>0.6</td>
<td>1.1447</td>
</tr>
<tr>
<td>0.8</td>
<td>1.1270</td>
</tr>
<tr>
<td>1.0</td>
<td>1.1154</td>
</tr>
<tr>
<td>1.2</td>
<td>1.0925</td>
</tr>
</tbody>
</table>

Along this direction there is a maximum in the center, so charge \( Q \) is not trapped.
4) **Problem 3.9**

Place a second image charge, \( q'' \), at the center of the sphere; this will not alter the fact that the sphere is an equipotential, but merely increase that potential from zero to \( V_0 = \frac{1}{4\pi \varepsilon_0 R^3} \).

\[
q'' = 4\pi \varepsilon_0 V_0 R \text{ at center of sphere.}
\]

For a neutral sphere, \( q' + q'' = 0 \).

\[
\begin{align*}
F &= \frac{\lambda}{4\pi \varepsilon_0} \left( q'' \left( a^2 + \frac{q'}{(a-b)^2} \right) = \frac{qq'}{4\pi \varepsilon_0} \left( -\frac{1}{a^2} + \frac{1}{(a-b)^2} \right) \right) \\
&= \frac{qq'}{4\pi \varepsilon_0} \frac{b(2a-b)}{a^2(a-b)^2} = q(-Rq/a) (2R/a) (2a - R^2/a) \\
&= -\frac{q^2}{4\pi \varepsilon_0} \left( \frac{R}{a} \right)^3 \left( 2a^2 - R^2 \right) \frac{(a^2 - R^2)^2}{a^2(a-R^2/a)^2}.
\end{align*}
\]

(Drop the minus sign, because the problem asks for the force of attraction.)

---

5) **Problem 3.10**

(a) Image problem: \( \lambda \) above, \(-\lambda \) below. Potential was found in Prob. 2.52:

\[
V(y, z) = \frac{2\lambda}{4\pi \varepsilon_0} \ln\left( \frac{s_-}{s_+} \right) = \frac{\lambda}{4\pi \varepsilon_0} \ln\left( \frac{y^2 + (z + d)^2}{y^2 + (z - d)^2} \right)
\]

(b) \( \sigma = -\varepsilon_0 \frac{\partial V}{\partial n} \). Here \( \frac{\partial V}{\partial n} = \frac{\partial V}{\partial z} \), evaluated at \( z = 0 \).

\[
\begin{align*}
\sigma(y) &= -\varepsilon_0 \frac{\lambda}{4\pi \varepsilon_0} \left\{ \frac{1}{y^2 + (z + d)^2} \frac{d}{2} (z + d) - \frac{1}{y^2 + (z - d)^2} \frac{d}{2} (z - d) \right\} \bigg|_{z=0} \\
&= -\frac{\lambda d}{4\pi} \left\{ \frac{d}{y^2 + d^2} - \frac{-d}{y^2 + d^2} \right\} = \frac{\lambda d}{\pi (y^2 + d^2)}.
\end{align*}
\]

Check: Total charge induced on a strip of width \( l \) parallel to the \( y \) axis:

\[
q_{ind} = -\frac{\lambda d}{\pi} \int_{-\infty}^{\infty} \frac{1}{y^2 + d^2} dy = -\frac{\lambda d}{\pi} \left[ \tan^{-1} \left( \frac{y}{d} \right) \right]_{-\infty}^{\infty} = -\frac{\lambda d}{\pi} \left[ \frac{\pi}{2} - \left( \frac{\pi}{2} \right) \right]
\]

\[
= -\lambda l. \text{ Therefore } q_{ind} = -\lambda, \text{ as it should be.}
\]

---

\[
\begin{align*}
V(y, z) &= -\frac{\lambda}{2\pi \varepsilon_0} \left[ \ln\left( \frac{s_+}{a} \right) - \ln\left( \frac{s_-}{a} \right) \right] \\
&= -\frac{\lambda}{2\pi \varepsilon_0} \left[ \ln\left( \frac{s_+}{a} \right) + \ln \left( \frac{s_+}{a} \right) - \ln\left( \frac{s_-}{a} \right) - \ln\left( \frac{s_-}{a} \right) \right] \\
&= \frac{\lambda}{2\pi \varepsilon_0} \ln\left( \frac{s_-}{s_+} \right)
\end{align*}
\]