Due date: Wednesday, Sept. 19

Griffiths reading:
- Review 1.2.1-4 and 1.3.1-2, and study 1.3.4 (divergence theorem)
- 2.2.1-2 (Gauss’s Law)
- 1.5 (Delta functions)

1. [4 points] (a) Compute the divergence of \( \mathbf{v}(\mathbf{r}) = 2xz \hat{x} + z^2 \hat{y} + (2yz + x^2) \hat{z} \).
   (b) Check the divergence theorem for \( \mathbf{v}(\mathbf{r}) \), taking the volume \( V \) to be a cubic box with one corner at the origin and the other at (111), i.e., extending over \( 0 < x < 1 \), \( 0 < y < 1 \), and \( 0 < z < 1 \). That is, compute the volume integral of \( \nabla \cdot \mathbf{v} \) and the surface integral of \( \mathbf{v} \cdot \hat{n} \) and check that they agree.

2. [2 points] Compute the divergence of \( \mathbf{v}(\mathbf{r}) = \hat{r}/r^n \) for arbitrary \( n \). (For now, don’t worry about what happens at \( r = 0 \); we will discuss it later.)

3. [4 points] A sphere of radius \( R \) is centered on the origin. The “northern” hemisphere defined by \( z > 0 \) carries a uniform surface charge \( \sigma \), while the “southern” hemisphere carries no charge. Find the electric field at the origin. (Use symmetry to argue that it is enough to calculated \( \hat{z} \cdot \mathbf{E} \), and then compute this by integrating over the hemisphere.)

4. [6 points] An infinitely long line charge of uniform density \( \lambda \) lies along the \( z \) axis, piercing a sphere of radius \( R \) centered on the origin. According to Eq. (2.9), the electric field is then
   \[
   \mathbf{E} = \frac{1}{2\pi \varepsilon_0} \frac{\lambda}{s} \hat{s}
   \]
   in cylindrical coordinates.
   (a) Calculate the electric flux passing out through the surface of the sphere.
   **Hint:** Show that \( \hat{E} \cdot \hat{r} = \sin \theta \), and then calculate the flux by working in spherical coordinates.
   (b) Does this agree with the expectation based on Gauss’s Law?

5. [4 points] Griffiths 1.44(a-d) and 1.45(a) (evaluate integrals with delta functions).