Question:
A sphere of radius $R$ contains a positive uniform interior charge density $\rho_0$, and the surface is covered with a uniform negative surface charge density that is precisely chosen to make the entire sphere net neutral, so that the total charge on the sphere is zero. By spherical symmetry the electric field is radial, $E(\mathbf{r}) = E(r) \hat{\mathbf{r}}$. Using Gauss’s law, find $E(r)$ both inside and outside the sphere.

Solution:

For neutrality, $\frac{4\pi R^3}{3} \rho_0 + 4\pi R^2 \sigma_0 = 0$

So $\sigma_0 = -\frac{\rho_0 R}{3}$, but this result is actually not needed.

$r > R$:

$$\oint E \cdot d\mathbf{a} = \frac{1}{\varepsilon_0} Q_{\text{enc}}$$

$$4\pi r^2 E(r) = \frac{1}{\varepsilon_0} Q_{\text{enc}}$$

Net neutral!

$$E(r) = 0, \quad r > R$$

$r < R$:

$$\oint E \cdot d\mathbf{a} = \frac{1}{\varepsilon_0} Q_{\text{enc}}$$

$$4\pi r^2 E(r) = \frac{1}{\varepsilon_0} + \frac{4\pi r^3}{3} \rho_0$$

$$E(r) = \frac{\rho_0}{3\varepsilon_0} r, \quad r < R$$