Ground rules:

- Open book
- Closed notes
- You may consult one page (both sides) of handwritten notes
- Write your answer directly on this sheet (continue onto back if necessary)

There are four questions of 25 points each. Pace yourself accordingly.

If you know the formulas for “standard cases” (e.g., the electric field or potential a certain distance from a point or line or plane charge), you may use these results without derivation unless the problem specifically asks you to derive it.

Partial credit will be given. Do as many parts of a problem as possible. If you are stuck on the first part of a problem, you should still try to say something about the later parts if possible.

Be sure to check that you have done all parts of all questions.

Feel free to raise your hand to ask a question.

Good luck!!

Legendre polynomials:

\[ P_0(u) = 1 \]
\[ P_1(u) = u \]
\[ P_2(u) = \frac{(3u^2 - 1)}{2} \]
\[ P_3(u) = \frac{(5u^3 - 3u)}{2} \]

Multipole expansion:

\[ V_n(r, \theta) = \frac{1}{4\pi \epsilon_0} I_n \frac{1}{r^{n+1}} P_n(\cos \theta) \quad , \quad I_n = \int r^n P_n(\cos \theta) \rho(r) \, d\tau \]
Problem 1. (25 points)

(a) The figure at right shows two concentric conducting spherical shells (hashed regions); the inner one extends from $R_1$ to $R_2$ and the outer one goes from $R_3$ to $R_4$. White spaces are vacuum. Find the capacitance between the two conductors. (Make sure your answer is written in terms of the correct radii.)

(b) Now interpreting the figure as showing two concentric conducting *cylindrical* shells in cross section, find the capacitance per unit length of this arrangement.
Problem 2. (25 points)

A positively charged styrofoam ball of radius $R$ carries a total charge $Q_0$ distributed uniformly through its interior (i.e., $Q_0 = \rho_0(4\pi R^3/3)$, but please express all answers in terms of $Q_0$, not $\rho_0$.) Its center is a distance $R + d$ below a conducting ceiling. (Note that the ball is not conducting.)

(a) Find the induced surface charge density $\sigma$ on the ceiling at point P just above the center of the ball.

(b) Find the electric field (magnitude and direction) at the center of the sphere.

(c) The weight of the ball is $mg$. How large must $Q_0$ be in order that the ball will stick to the ceiling (i.e., with $d = 0$)?
You are given that the potential $V(r, \theta) = V_0 \cos(\theta)$
at an inner radius $r = a$, and it vanishes ($V = 0$) at an outer radius $r = 2a$. Also, you know that there is no charge filling the shaded vacuum between these two radii. Find $V(r, \theta)$ in this shaded region, i.e., for $a < r < 2a$. 

**Problem 4.** (25 points)

Two rings of radius $a$ are centered around the $z$ axis with their centers at $z = b$ and $z = -b$.

(a) The top and bottom rings carry linear charge density $\lambda_0$ and $-\lambda_0$ respectively. What is the leading behavior of $V(r, \theta)$ far away?

(b) Now both rings carry the same linear charge density $\lambda_0$. Answer the same question again for this case.

(c) Suppose you are asked to modify the charge configuration of part (b) by adding one point charge $Q$ at a location of your choosing, in such a way that the leading behavior of $V$ far away is the quadrupolar one that decays as $1/r^3$. What would you choose for $Q$, and where would you place this charge? Explain.