Physics 385 **Final Exam (Make-Up)** **Dec. 20, 1999**

Name: ________________________________

Ground rules:

- Open book
- Closed notes
- You may consult one page (two sides) of handwritten notes
- A calculator is allowed but will not be useful
- Write your answer directly on these sheets (continue onto back, if necessary)

There are 8 questions; point assignments are given for each, for a total of 100 points. Do all problems. Pace yourself appropriately.

Show your reasoning. A correct answer all by itself, without any indication of the reasoning leading to it, will not get full credit. However, if you may use the formulas for "standard cases" (for example, the electric field or potential a certain distance from a point or line or plane charge) in your solution without deriving them.

Partial credit will be given. Do as many parts of a problem as possible. If you are stuck on the first part of a problem, you should still try to say something about the later parts if possible.

Be sure to check that you have done all parts of all questions.

Feel free to raise your hand to ask a question.

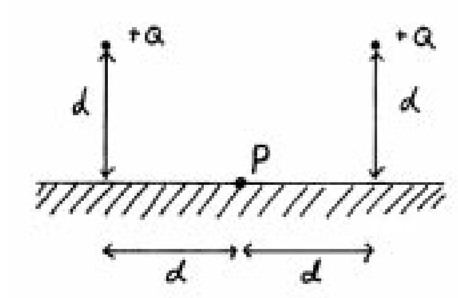
Good luck, and happy holidays!

Formulas for Legendre polynomials:

$P_0(x) = 1$
$P_1(x) = x$
$P_2(x) = (3x^2 - 1)/2$
$P_3(x) = (5x^3 - 3x)/2$

Problem 1 (15 points)

Two point charges $+Q$ are each located a distance d above the infinite planar surface of a grounded conductor, a distance $2d$ apart. The point P lies on the planar surface midway between the charges as shown.



a) Find V at P.

b) Find $\mathbf{E}$ (magnitude and direction) at P.

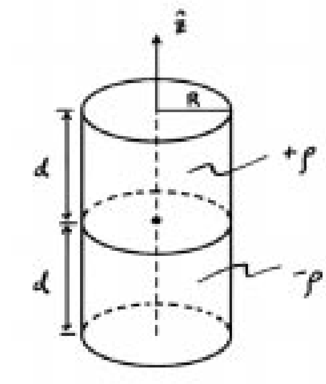
c) Find the induced suface charge density σ at P.

Problem 2 (10 points)

Consider the hemispherical volume defined by $r \leq R$ and $\theta \leq \pi/2$, in standard (r, θ, φ) spherical coordinates. There is no charge inside this hemispherical volume, and the electric potential is $V = 0$ on the planar bottom surface ($z = 0$), and $V(R, \theta) = V_0 P_3(\cos\theta) = V_0\,(5\cos^3\theta - 3\cos\theta)/2$ on the hemispherical top surface ($r = R$). Find $V(r, \theta)$ inside the region.

[Hint: Look for a separable form in spherical coordinates.]

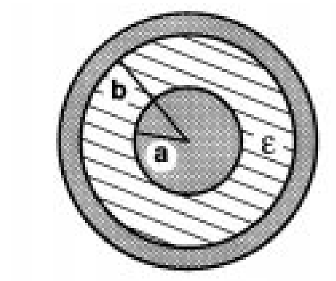
Problem 3 (10 points)



A cylinder of radius R and height $2d$ is centered on the origin and has its axis along z. The top and bottom halves of the cylinder are filled with uniform volume charges $+\rho$ and $-\rho$ respectively, as shown. Find the leading term in the multipole expansion of the potential $V(r, \theta)$ far from the cylinder.

Problem 4 (15 points)

A linear dielectric material of dielectric permittivity ϵ fills the space between a metallic sphere of radius a and a metallic shell of inner radius b.



a) Find the electric field $E(r)$ that appears in the dielectric region, $a < r < b$, if free charges $+Q_f$ and $-Q_f$ are placed on the inner and outer conductors, respectively.

b) Under the same conditions as in (a), find the electric potential difference between the inner and outer conductors.

c) What is the capacitance C of this system?

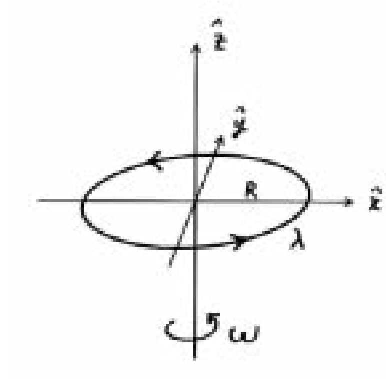
<u>Problem 5</u> (10 points)

A plane slab of thickness $2d$ extends infinitely in the x and y directions and is centered on the origin so that it fills the region $-d \leq z \leq d$. The material making up the slab has polarization $\mathbf{P} = A\,z\,\hat{z}$. No free charge is present.

a) Locate and calculate all bound charges, both of type ρ_b and σ_b.

b) Find the electric field $E(z)$, both inside ($|z| < d$) and outside ($|z| > d$) the slab.

Problem 6 (15 points)



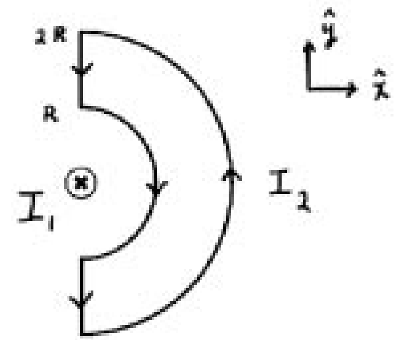
A thin circular ring of radius R lying in the $x - y$ plane carries a linear charge density λ and is spinning about its center at angular frequency ω as shown, so that the charge is moving at speed $v = R\omega$.

a) Find the electric potential V at the center of the ring.
b) Find the electric field $\mathbf{E}$ at the center of the ring.
c) Find the magnetic field $\mathbf{B}$ at the center of the ring.

Problem 7 (10 points)

An infinite straight wire of radius a carries a current in the $+\hat{x}$ direction. The current density $J(s)$ is not uniform, but depends on the distance s from the center of the wire as $J(s) = \beta/s$, where β is a constant. Find the magnetic field (magnitude and direction) both inside and outside the wire.

Problem 8 (15 points)



One infinite straight wire carries a current I_1 into the page (i.e., in the $-\hat{z}$ direction), while a second wire forms a current loop lying in the $x-y$ plane and carries a current I_2 as shown. (The inner and outer radii are R and $2R$, respectively, for the semicircular portions of the current loop.) Find the total force (magnitude and direction) acting on the current loop I_2 due to the magnetic field of I_1.

Scrap paper – Page 1

Scrap paper – Page 2