Physics 385 **Final Exam** **Dec. 21, 2000**

Name: ______________________________

Ground rules:

- Open book
- Closed notes
- You may consult one page (two sides) of handwritten notes
- A calculator is allowed but will not be useful
- Write your answer directly on these sheets (continue onto back, if necessary)

There are 8 questions; point assignments are given for each, for a total of 100 points. Do all problems. Pace yourself appropriately.

Show your reasoning. A correct answer all by itself, without any indication of the reasoning leading to it, will not get full credit. However, you may use the formulas for "standard cases" (for example, the electric field or potential a certain distance from a point or line or plane charge) in your solution without deriving them.

Partial credit will be given. Do as many parts of a problem as possible. If you are stuck on the first part of a problem, you should still try to say something about the later parts if possible.

Be sure to check that you have done all parts of all questions.

Feel free to raise your hand to ask a question.

Good luck, and happy holidays!

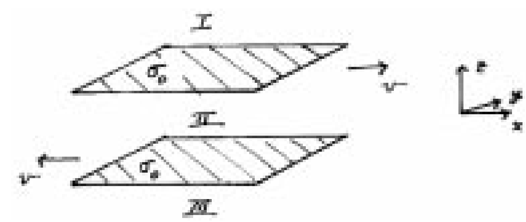
Legendre polynomials:

$$P_0(x) = 1, \quad P_1(x) = x, \quad P_2(x) = (3x^2 - 1)/2, \quad P_3(x) = (5x^3 - 3x)/2$$

Multipole expansion:

$$V(r,\theta) = \frac{1}{4\pi\epsilon_0} \sum_{n=0}^{\infty} I_n \, \frac{1}{r^{n+1}} \, P_n(\cos\theta) \ , \qquad I_n = \int r^n \, P_n(\cos\theta) \, \rho(\mathbf{r}) \, d\tau$$

Problem 1 (15 points)



Two infinite planes of areal charge density σ_0 are located at $z = d$ and $z = -d$. Let the regions above, between, and below the planes be regions I, II, and III, respectively.

a) Find the electric field (magnitude and direction) in all three regions.

b) The top plane is moving in the $+\hat{x}$ direction at speed v, while the bottom plane is moving in the opposite ($-\hat{x}$) direction at the same speed v. Find the magnetic field (magnitude and direction) in all three regions.

c) Check that your solutions to part (a) and (b) obey the boundary conditions expected for $\Delta E_\perp$, $\Delta \mathbf{E}_\parallel$, $\Delta B_\perp$, and $\Delta \mathbf{B}_\parallel$ at the top planar boundary between I and II.

<u>Problem 2</u> (10 points)

A dielectric sphere of radius R has electric polarization $\mathbf{P} = A\,r^2\,\hat{r}$, where A is some constant. No free charge is present.

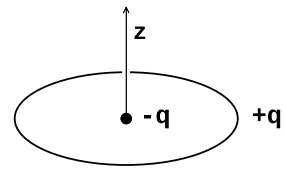
a) Find all bound charges, both of type ρ_b and σ_b.

b) Check overall neutrality.

c) Find the (radial) electric field $E(r)$, both for $r < R$ and $r > R$.

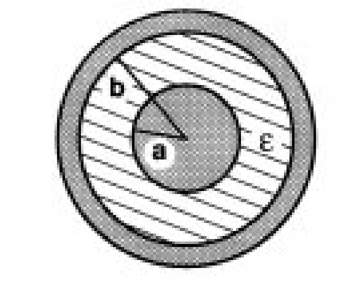
<u>Problem 3</u> (10 points)

A circular ring of radius a lies in the x-y plane, centered on the origin. It carries a total positive charge q that is uniformly distributed over its circumference, and there is also a negative point charge $-q$ at the center of the ring (i.e., at the origin).



Find the leading term in the multipole expansion of the potential $V(r, \theta)$ far from this system of charges.

<u>Problem 4</u> (15 points)

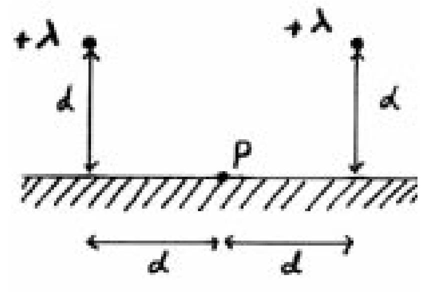


A linear dielectric material of dielectric permittivity ϵ fills the space between an infinite cylindrical wire of radius a and an outer conducting cylinder of inner radius b, as shown. Assume that there is a total free charge per unit length λ_f on the inner cylinder, and similarly $-\lambda_f$ on the outer cylinder.

a) Find $\mathbf{E}(s)$, $\mathbf{D}(s)$, and $\mathbf{P}(s)$ in the dielectric region, $a < s < b$.

b) What is the capacitance per unit length (C/L) of this system?

Problem 5 (15 points)



Two infinite line charges of linear density λ are parallel to each other at a distance d above an infinite grounded conducting plate. They are a distance $2d$ apart, as shown *in cross section* in the figure. The point P lies on the planar surface midway between the line charges as shown.

a) Find V at P.

b) Find $\mathbf{E}$ (magnitude and direction) at P.

c) Find the induced suface charge density σ at P.

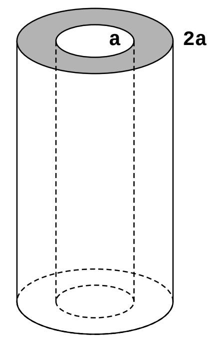
Problem 6 (10 points)

Suppose you know that $V(\theta, \phi) = V_0\, P_3(\cos\theta)$ on the surface of a sphere of radius R, and that no charges are present except on this surface. (That is, V satisfies Laplace's Equation for $r < R$ and for $r > R$. Moreover, $V \to 0$ as $r \to \infty$ as usual.)

a) Find $V(\mathbf{r})$ everywhere.

b) Describe quantitatively the charge distribution that is present on the surface.

Problem 7 (15 points)



An infinite hollow cylindrical wire has inner radius a and outer radius $2a$ as shown, and carries a total current I running in the $+z$ direction. The current is uniformly distributed across the cross-sectional area of the wire. Using Ampère's Law:

a) Find $\mathbf{B}$ inside the hollow region ($s < a$).

b) Find $\mathbf{B}$ ouside the wire ($s > 2a$).

c) Show that $J = I/3\pi a^2$ and then find $\mathbf{B}(s)$ in the current-carrying region $a < s < 2a$.

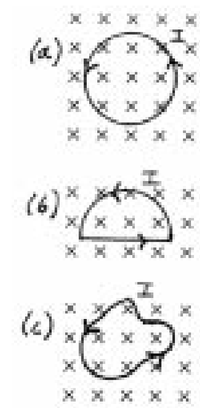
Problem 8 (10 points)

A uniform magnetic field is directed into the plane of the paper, $\mathbf{B} = -B_0\,\hat{z}$. A current loop lies in the $z = 0$ plane perpendicular to the field. Show that the *total force* on the loop is zero:

a) For a circular loop.

b) For a semicircular loop.

c) Extra credit (+5): For a planar loop of arbitrary shape!



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