

Name: \_\_\_\_\_

Ground rules:

- Open book
- Closed notes
- You may consult one page (two sides) of handwritten notes
- A calculator is allowed but will not be useful
- Write your answer directly on these sheets (continue onto back, if necessary)

**There are four questions of 25 points each. Pace yourself accordingly.**

If you know the formulas for “standard cases” (e.g., the electric field or potential a certain distance from a point or line or plane charge), you may use these results without derivation unless the problem specifically asks you to derive it.

Partial credit will be given. Do as many parts of a problem as possible. If you are stuck on the first part of a problem, you should still try to say something about the later parts if possible.

Be sure to check that you have done all parts of all questions.

Feel free to raise your hand to ask a question.

Good luck!!

Formulas for Legendre polynomials:

$$P_0(x) = 1$$

$$P_1(x) = x$$

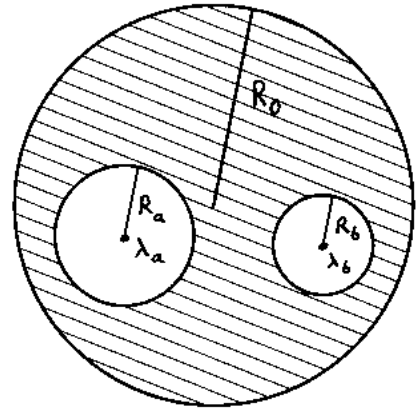
$$P_2(x) = (3x^2 - 1)/2$$

$$P_3(x) = (5x^3 - 3x)/2$$

**Problem 1** (25 points)

The figure shows a cross section of a conducting *cylinder* of radius  $R_0$ . Two cylindrical cavities of radii  $R_a$  and  $R_b$  have been drilled parallel to the axis of the cylinder, and line charges  $\lambda_a$  and  $\lambda_b$  are centered in these cavities, respectively. (Take the entire arrangement to be infinitely long.)

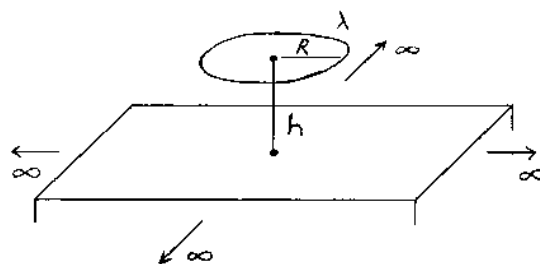
- a) Find the surface charges  $\sigma_a$  and  $\sigma_b$  that appear on the inner surfaces of the two cavities.
- b) Find the surface charge  $\sigma_0$  that appears on the outer surface of the conductor.



**Problem 2** (25 points)

A circular wire ring of radius  $R$  carries a uniform line charge  $\lambda$ , and lies in a plane that is parallel to the surface of an (infinite) grounded conducting plate. The distance between the ring and the surface of the plate is  $h$ .

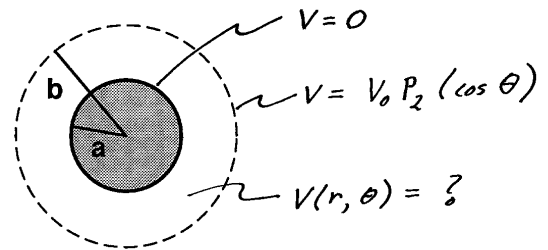
- a) Find the potential  $V$  at the center of the wire ring.
- b) Find the induced surface charge  $\sigma$  at the point on the metal surface directly under the center of the ring.



**Problem 3** (25 points)

A conducting sphere of radius  $a$  is centered at the origin. You know only that the conducting sphere is grounded ( $V = 0$ ); that the space between  $a < r < b$  contains no charge; and that the potential on the spherical surface at  $r = b$  is  $V(r, \theta, \phi)|_{r=b} = V_0 P_2(\cos \theta)$ , independent of  $\phi$ . ( $P_2$  is the Legendre polynomial  $P_n$  with  $n=2$ .) Find the potential  $V(r, \theta)$  in between the spherical surfaces (i.e., for  $a < r < b$ ).

Hint: Don't be surprised if your answer contains fifth powers of some quantities.



**Problem 4** (25 points)

A cylinder of radius  $R$  and length  $L$  is centered on the origin with its axis along  $\hat{x}$ . The surface cap at  $x = +L/2$  has uniform surface charge  $+\sigma$ , and the other cap at  $x = -L/2$  has surface charge  $-\sigma$ . Find the leading behavior of the potential far from the cylinder.

