

Physics 385 Fall 2007 Exam 1 Review

2.1 The Electric Field: Force between two charges; Coulomb's Law; $\mathbf{r} = \mathbf{r} - \mathbf{r}'$; Superposition; Definition of the Electric Field; Electric Field of a point charge; Electric Field from continuous charge distribution; $dq \rightarrow \lambda dl \sim \sigma da \sim \rho d\tau$.

2.2 Div and Curl of E: Field lines, rules, representation of Electric field strength, Sketch; Flux of Electric Field; Gauss's Law : $\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$; Continuous charge distribution : $\oint_S \vec{E} \cdot d\vec{a} = \int_V \frac{\rho}{\epsilon_0} d\tau$; Divergence Theorem: $\oint_S \vec{E} \cdot d\vec{a} = \int_V (\vec{\nabla} \cdot \vec{E}) d\tau$; Local version of Gauss's Law: $\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$; Gauss's Law to get the Electric field from: a spherically symmetric charge distribution, a cylindrically symmetric charge distribution, a planar charge distribution; $\int \vec{E} \cdot d\vec{l} = 0 \rightarrow \vec{\nabla} \times \vec{E} = 0$

2.3 The Electrostatic Potential; $V(\vec{r}) = - \int_{ref}^{\vec{r}} \vec{E} \cdot d\vec{l}$; $V(b) - V(a) = - \int_a^b \vec{E} \cdot d\vec{l}$;

$\vec{E} = -\vec{\nabla}V$; Take care of reference point; typically $ref = \infty$, where $V(\infty) = 0$, except when charge distribution is not bounded (e.g., inf plane of charge); $V(\mathbf{b}) - V(\mathbf{a})$ is unique; $V(\mathbf{r})$ is unique up to a constant; V obeys superposition ($V = V_1 + V_2 + \dots$); Should know or be able to quickly derive potential of a spherically symmetric charge distribution, a cylindrically symmetric charge distribution, and a planar charge distribution. Laplace's Equation: $\nabla^2 V = 0$ in region of space where charge density is zero. In general, have Poisson's

Equation: $\nabla^2 V = -\frac{\rho}{\epsilon_0}$; $V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}')}{|\vec{r}-\vec{r}'|} d\tau'$; $V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\sigma(\vec{r}')}{|\vec{r}-\vec{r}'|} da'$;

$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda(\vec{r}')}{|\vec{r}-\vec{r}'|} dl'$; Boundary Conditions: $\vec{E}_{above}^\perp - \vec{E}_{below}^\perp = \frac{\sigma}{\epsilon_0}$; $\vec{E}_{above}^\parallel = \vec{E}_{below}^\parallel$;

$V_{above} = V_{below}$

2.4 Work and Energy: For an external agent to move point charge from a to b in presence of electric field: $W = \int_a^b \vec{F} \cdot d\vec{l} = -Q \int_a^b \vec{E} \cdot d\vec{l} = Q[V(b) - V(a)]$; with the reference point at infinity, we have $W = QV(\vec{r})$; Work to bring point charge, Q , from infinity to

the point \mathbf{r} near an existing array of N point charges: $W = Q \sum_{i=1}^N \frac{1}{4\pi\epsilon_0} \frac{q_i}{|\vec{r}-\vec{r}_i|}$; Work to assemble

an array of N point charges: $W = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^N \sum_{j>i}^N \frac{q_i q_j}{|\vec{r}_i - \vec{r}_j|} = \frac{1}{8\pi\epsilon_0} \sum_{i=1}^N \sum_{j \neq i}^N \frac{q_i q_j}{|\vec{r}_i - \vec{r}_j|} =$

$\frac{1}{2} \sum_{i=1}^N q_i \left(\sum_{j \neq i}^N \frac{1}{4\pi\epsilon_0} \frac{q_j}{|\vec{r}_i - \vec{r}_j|} \right) = \frac{1}{2} \sum_{i=1}^N q_i V(\vec{r}_i)$. For continuous charge distributions:

$W = \frac{1}{2} \int \rho V d\tau = \frac{\epsilon_0}{2} \left[- \int \vec{E} \cdot (\vec{\nabla} V) d\tau + \oint V \vec{E} \cdot d\vec{a} \right] = \frac{\epsilon_0}{2} \left[\int E^2 d\tau + \oint V \vec{E} \cdot d\vec{a} \right]$, or if the surface extends to infinity, $W = \frac{\epsilon_0}{2} \int_{all\ space} E^2 d\tau$. Energy density of electric field = $\frac{\epsilon_0}{2} E^2$.

2.5 Conductors: $E = 0$ inside conductor (view this as caused by the rearrangement of free charges on surface of conductor just so that contribution to field from each is exactly cancelled throughout the volume of the conductor from the rest of the charges).

$\rho = 0$ inside conductor: $\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$ but since $E = 0$ everywhere, $\vec{\nabla} \cdot \vec{E} = 0$ so $\rho = 0$.

Net charge on surface of conductor.

A conductor is an equipotential: $V(b) - V(a) = - \int_a^b \vec{E} \cdot d\vec{l}$ but since $E=0$ inside, then $V(\mathbf{b}) = V(\mathbf{a})$.

Charge distributions near conductors will induce a surface charge density. If the net charge on the conductor is zero, the net induced charge on the conductor is zero, but a nonuniform surface charge distribution may occur. As $E = 0$ in the body of a conductor, cavities within the conductor that contain net charge will induce net charge on surface of cavity. $E = 0$ in cavity that contains no net charge. If there is a net charge within the volume bound by the outer surface of the conductor, an equal charge will reside on the outer surface of the conductor. That charge will be uniformly distributed if the outer surface is symmetric. When a charge $+Q$ is transferred from one initially neutral conductor

to another, a potential difference $V = V_+ - V_- = - \int_{(-)}^{(+)} \vec{E} \cdot d\vec{l}$ is established between the two

conductors. The potential difference developed by the transfer of a given charge Q depends on the geometry of the conductors. The capacitance is defined as $C = Q/V$. For parallel plate capacitor, $C = A\epsilon_0/d$ where A is area of plates and d is the plate separation. Know other simple geometries (concentric spheres, coaxial cylinders) as well. The differential of work done to move charge dq from one plate to another is $dW = Vdq = (q/C)dq$ therefore the work to charge a capacitor up to a total charge Q is:

$$W = \int_0^Q \frac{q}{C} dq = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} CV^2.$$

3.1: Laplace's Equation: $\nabla^2 V = 0$. Special case of Poisson's Equation: $\nabla^2 V = -\frac{\rho}{\epsilon_0}$.

Know expression in Cartesian coordinates: $\nabla^2 V = \frac{\partial^2 V_x}{\partial x^2} + \frac{\partial^2 V_y}{\partial y^2} + \frac{\partial^2 V_z}{\partial z^2} = 0$. Know how to apply the equation using spherical and cylindrical coordinates. Special properties of the solutions to Laplace's equation: $V(\mathbf{r})$ is average of value of V in sphere centered at \mathbf{r} ; no local max or min in volume, only on boundary. Understand uniqueness theorems: (1) Solution to Laplace's equation is uniquely determined in volume \mathcal{V} bounded by surface \mathcal{S} , if V is specified on \mathcal{S} ; (2) In a volume \mathcal{V} that contains charge density ρ and is surrounded by conductors containing charge Q_i , then E is uniquely determined if all Q_i are specified and entire volume is surrounded by a conductor or extends to infinity.