Physics 385
Electromagnetism

Fall 2006 - Prof. Bartynski

Exam II

Wednesday, 15-November-2006
Noon – 1:20 PM

Closed Book. Closed Notes.
Calculator OK, Two Cheat Sheets OK.

Do not open this exam until instructed to do so.
Please fill out the information on the cover of your blue book.
Answer all 4 problems.

Possibly useful information:

$P_o(\cos\theta) = 1$;

$P_1(\cos\theta) = \cos\theta$;

$P_2(\cos\theta) = \frac{1}{2}(3\cos^2\theta - 1)$;

$P_3(\cos\theta) = \frac{1}{2}(5\cos^3\theta - 3\cos\theta)$

$$\int_0^\pi P_l(\cos\theta)P_{l'}(\cos\theta)\sin\theta d\theta = \tfrac{2}{2l+1}\delta_{l,l'} \quad ; \quad \int_0^a \sin(n\pi x/a)\sin(n'\pi x/a)dx = \tfrac{a}{2}\delta_{n,n'}$$

$$\cos 2\theta = (2\cos^2\theta - 1) \quad ; \quad \sin^3\theta = \tfrac{3}{4}\sin\theta - \tfrac{1}{4}\sin(3\theta)$$

$$\int \frac{dx}{(x^2+a^2)^{\frac{1}{2}}} = \ln[x + (x^2+a^2)^{\frac{1}{2}}]$$

$$\int \frac{xdx}{(x^2+a^2)^{\frac{1}{2}}} = (x^2+a^2)^{\frac{1}{2}}$$

$$\int \frac{x^2dx}{(x^2+a^2)^{\frac{1}{2}}} = \tfrac{x}{2}(x^2+a^2)^{\frac{1}{2}} - \tfrac{a^2}{2}\ln[x+(x^2+a^2)^{\frac{1}{2}}]$$

$$\int \frac{x^3dx}{(x^2+a^2)^{\frac{1}{2}}} = \tfrac{1}{3}(x^2+a^2)^{\frac{1}{2}} - a^2(x^2+a^2)^{\frac{1}{2}}$$

$$\int \frac{dx}{(x^2+a^2)^{\frac{3}{2}}} = \frac{x}{a^2(x^2+a^2)^{\frac{1}{2}}}$$

$$\int \frac{xdx}{(x^2+a^2)^{\frac{3}{2}}} = \frac{-1}{(x^2+a^2)^{\frac{1}{2}}}$$

$$\int \frac{x^2dx}{(x^2+a^2)^{\frac{3}{2}}} = \frac{-x}{(x^2+a^2)^{\frac{1}{2}}} + \ln[x+(x^2+a^2)^{\frac{1}{2}}]$$

$$\int \frac{x^3dx}{(x^2+a^2)^{\frac{3}{2}}} = (x^2+a^2)^{\frac{1}{2}} + \frac{a^2}{(x^2+a^2)^{\frac{1}{2}}}$$

$$\int x\cos(x)dx = \cos(x) + x\sin(x)$$

$$\int x^2\cos(x)dx = 2x\cos(x) + (x^2-2)\sin(x)$$

Binomial expansion: $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots$ where $x^2 < 1$

Some useful constants: $e = 1.6 \times 10^{-19}$ C; $\varepsilon_o = 8.85 \times 10^{-12}$ C^2/Nm^2 ; $\mu_o = 4\pi \times 10^{-7}$ N/A^2

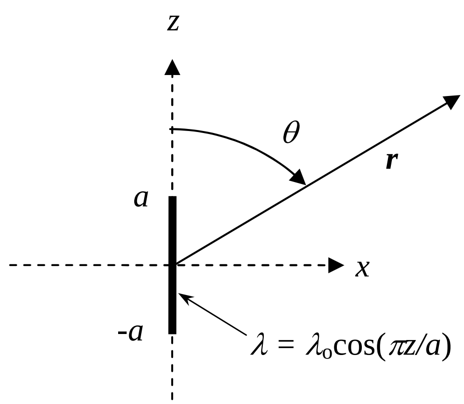
Law of Cosines: $\mathfrak{R} = [r^2 + (r')^2 - 2rr'\cos\theta]^{1/2}$ where $\mathfrak{R} = |\mathbf{r} - \mathbf{r}'|$

25 pts 1) A radio antenna extends from $-a$ to a along the z-axis. At a certain instant in time, the linear charge distribution along the antenna is given by $\lambda = \lambda_o \cos(\pi z/a)$.

(a) Show that the monopole moment of this charge distribution is zero; *i.e.*, $Q = 0$.

(b) Show that the dipole moment is zero; *i.e.*, $\boldsymbol{p} = 0$.

(c) What is the quadrupole term of the potential at the point (r,θ)?



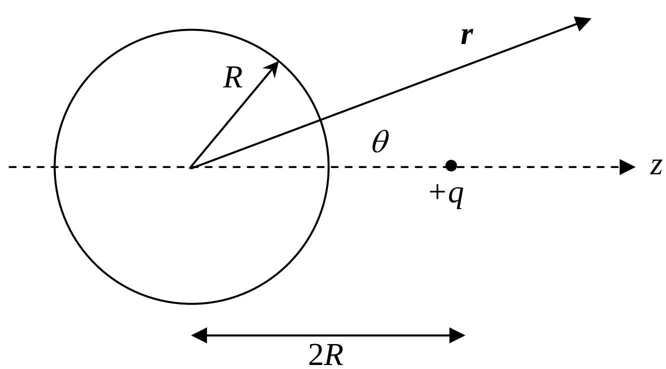
25 pts 2) A point charge, q, is at $z = 2R$ and a <u>*grounded*</u> (*i.e.*, $V = 0$) conducting sphere of radius R is centered at the origin.

(a) What is the location and value of the image charge needed to find the potential for $r > R$?

(b) Use the method of image charges to find the potential for $r > R$. Express your answer in terms of r, R and θ.

(c) What is the orientation of the Electric Field just outside the sphere?

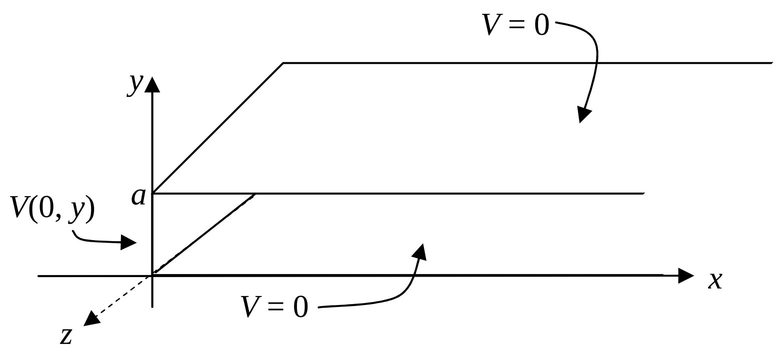
(d) Find an expression for the Electric Field just outside the sphere. (That is, calculate $\boldsymbol{E} = -\nabla V$ and evaluate this at $r = R$.)



25 pts 3) Consider two grounded conducting sheets, one lying in the *x*-*y* plane, the other parallel to it and distance *a* above it. The sheets extend indefinitely in the *z*-direction, and exist for $x > 0$. The potential along a strip at $x = 0$, $0 < y < a$, and all *z* is given by:

$$V(0, y) = V_o \sin^3(\pi y/a).$$

Use separation of variables to solve Laplace's equation and find the potential in the gap for $x > 0$. [HINT: Express $\sin^3(\theta)$ as a linear combination of $\sin(n\theta)$ with different *n*'s and by the orthogonality of the sin functions, you should find that only two terms of the general expansion survive.]



25 pts 4) A sphere of dielectric material has uniform polarization $\boldsymbol{P} = P\hat{\mathbf{z}}$

(a) Find the bound surface charge density, σ_b at $r = R$ as a function of θ.

(b) Consider the multipole expansion of the potential for $r > R$. Show that the monopole term is zero. Calculate the dipole term. In fact, for $r > R$, all other multipole terms are zero.

(c) In terms of an expansion in Legendre polynomials, write a general expression for the potential for $r < R$ (Your expression should contains only terms that <u>*do not diverge*</u> as r $\rightarrow 0$).

(d) Use continuity of the potential at $r = R$ (that is, $V(r_+, \theta)|_{r=R} = V(r_-, \theta)|_{r=R}$) to simplify the general expression in (c) and find a simple expression for the potential for $r < R$. [By appealing to the orthogonality property of the Legendre polynomials, you should find that only one term from (c) survives.]