

Physics 385
Electromagnetism

Fall 2006 - Prof. Bartynski

Exam I

Friday, 13-October-2006

1:40 PM – 3:00 PM

Closed Book. Closed Notes.
Calculator OK, One Cheat Sheet OK.

Do not open this exam until instructed to do so.
Please fill out the information on the cover of your blue book.
Answer all 4 problems.

Possibly useful information:

$$\int \frac{dx}{(x^2 + a^2)^{\frac{1}{2}}} = \ln[x + (x^2 + a^2)^{\frac{1}{2}}]$$

$$\int \frac{xdx}{(x^2 + a^2)^{\frac{1}{2}}} = (x^2 + a^2)^{\frac{1}{2}}$$

$$\int \frac{x^2 dx}{(x^2 + a^2)^{\frac{1}{2}}} = \frac{x}{2}(x^2 + a^2)^{\frac{1}{2}} - \frac{a^2}{2} \ln[x + (x^2 + a^2)^{\frac{1}{2}}] \quad \int \frac{x^3 dx}{(x^2 + a^2)^{\frac{1}{2}}} = \frac{1}{3}(x^2 + a^2)^{\frac{1}{2}} - a^2(x^2 + a^2)^{\frac{1}{2}}$$

$$\int \frac{dx}{(x^2 + a^2)^{\frac{3}{2}}} = \frac{x}{a^2(x^2 + a^2)^{\frac{1}{2}}}$$

$$\int \frac{xdx}{(x^2 + a^2)^{\frac{3}{2}}} = \frac{-1}{(x^2 + a^2)^{\frac{1}{2}}}$$

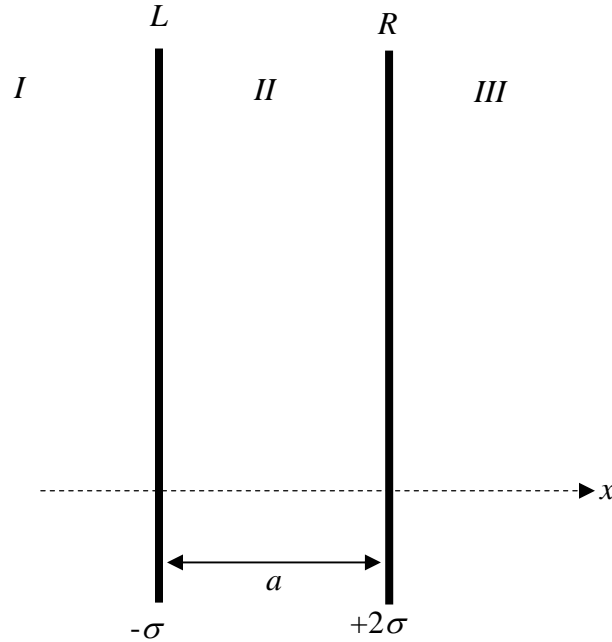
$$\int \frac{x^2 dx}{(x^2 + a^2)^{\frac{3}{2}}} = \frac{-x}{(x^2 + a^2)^{\frac{1}{2}}} + \ln[x + (x^2 + a^2)^{\frac{1}{2}}] \quad \int \frac{x^3 dx}{(x^2 + a^2)^{\frac{3}{2}}} = (x^2 + a^2)^{\frac{1}{2}} + \frac{a^2}{(x^2 + a^2)^{\frac{1}{2}}}$$

Binomial expansion: $(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots$ where $x^2 < 1$

Some useful constants: $e = 1.6 \times 10^{-19} \text{ C}$; $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2$

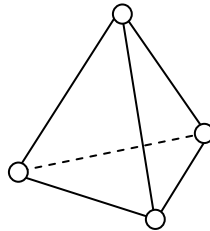
25 pts 1) Consider two infinite planar sheets of charge, perpendicular to the x -axis, separated by a distance a . The sheets are shown edge-on in the figure below. The sheet on the left (L) has uniform surface charge density $-\sigma_0$, while the sheet on the right (R) has $+2\sigma_0$.

- (a) What is the electric field (magnitude and direction) in each region of space I, II, and III ?
 (b) What is the force per unit area (magnitude and direction) exerted on sheet L ? On sheet R ?



25 pts 2) Suppose four ions are arranged as a tetrahedron, as shown below. Each ion is equidistant from the other three, *i.e.*, the edge of the tetrahedron is of length a .

- (a) Suppose two ions have charge $-q$ and two have charge $+q$, what is the energy stored in this configuration?
 (b) Is this configuration energetically more stable than the ions separated from each other at infinity?
 (c) Repeat parts (a) and (b) assuming one ion is $-q$ and the remaining three are $+q$.



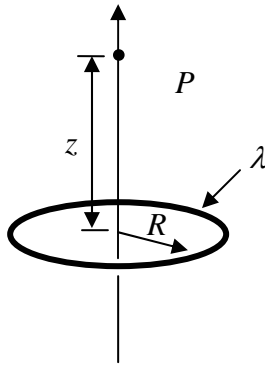
20 pts 3) In some region of space, the electric field is given by:

$$\vec{E} = \frac{ar^3}{\epsilon_0} \hat{r}$$

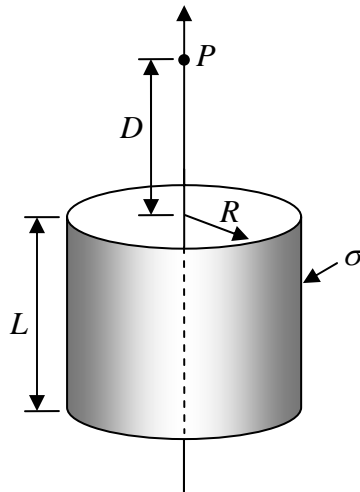
where a is a constant.

- (a) What is the charge density at a point a distance r from the origin?
 (b) What is the total charge enclosed in a sphere of radius R ?

30 pts 4) (a) Consider a uniform ring of charge with radius R and linear charge density λ . What is the electric field at point P which is a distance z above the center of the ring? [Use symmetry to think about the direction of the electric field at P .]



(b) Now consider a hollow tube of radius R and surface charge density σ and whose axis coincides with the z -axis. What is the electric field at point P which lies on a distance D above the top of the cylinder? [HINT: you may wish to think of the cylinder as a stack of charged rings of thickness dz and linear charge density $\lambda = \sigma dz$.]



EXTRA CREDIT (10 pts): Show that the solution for 4(b) reduces for the expected Coulomb field for $D \gg R, L$