Physics 385
Electromagnetism
Fall 2005 - Prof. Bartynski
Final Exam
Monday, 19-December-2005
8:00 AM – 11:00 AM

Closed Book. Closed Notes.
Calculator OK, Three Cheat Sheets OK.

Do not open this exam until instructed to do so.
Please fill out the information on the cover of your blue book(s)
Answer all 6 problems.

Possibly useful information:

$P_o(\cos\theta) = 1$; $P_2(\cos\theta) = \frac{1}{2}(3\cos^2\theta - 1)$;

$P_1(\cos\theta) = \cos\theta$; $P_3(\cos\theta) = \frac{1}{2}(5\cos^3\theta - 3\cos\theta)$

$$\int_0^\pi P_l(\cos\theta)P_{l'}(\cos\theta)\sin\theta d\theta = \frac{2}{2l+1}\delta_{l,l'} \quad ; \quad \int_0^a \sin(n\pi x/a)\sin(n'\pi x/a)dx = \frac{a}{2}\delta_{n,n'}$$

$$\cos 2\theta = (2\cos^2\theta - 1) \quad ; \quad \sin^3\theta = \frac{3}{4}\sin\theta - \frac{1}{4}\sin(3\theta)$$

$$\int \frac{dx}{(x^2+a^2)^{\frac{1}{2}}} = \ln[x + (x^2+a^2)^{\frac{1}{2}}] \qquad \int \frac{xdx}{(x^2+a^2)^{\frac{1}{2}}} = (x^2+a^2)^{\frac{1}{2}}$$

$$\int \frac{x^2dx}{(x^2+a^2)^{\frac{1}{2}}} = \frac{x}{2}(x^2+a^2)^{\frac{1}{2}} - \frac{a^2}{2}\ln[x+(x^2+a^2)^{\frac{1}{2}}] \qquad \int \frac{x^3dx}{(x^2+a^2)^{\frac{1}{2}}} = \frac{1}{3}(x^2+a^2)^{\frac{1}{2}} - a^2(x^2+a^2)^{\frac{1}{2}}$$

$$\int \frac{dx}{(x^2+a^2)^{\frac{3}{2}}} = \frac{x}{a^2(x^2+a^2)^{\frac{1}{2}}} \qquad \int \frac{xdx}{(x^2+a^2)^{\frac{3}{2}}} = \frac{-1}{(x^2+a^2)^{\frac{1}{2}}}$$

$$\int \frac{x^2dx}{(x^2+a^2)^{\frac{3}{2}}} = \frac{-x}{(x^2+a^2)^{\frac{1}{2}}} + \ln[x+(x^2+a^2)^{\frac{1}{2}}] \qquad \int \frac{x^3dx}{(x^2+a^2)^{\frac{3}{2}}} = (x^2+a^2)^{\frac{1}{2}} + \frac{a^2}{(x^2+a^2)^{\frac{1}{2}}}$$

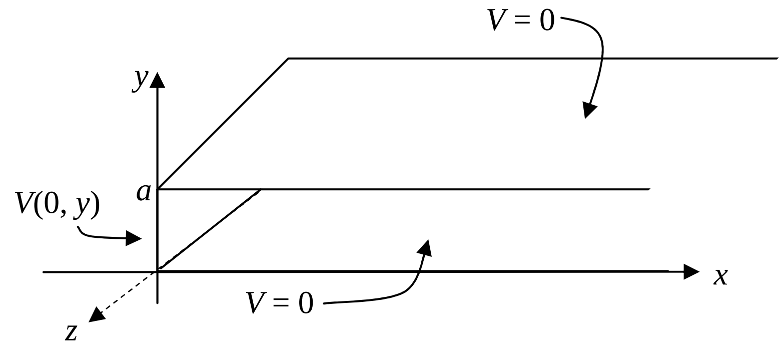
Binomial expansion: $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots$ where $x^2 < 1$

Some useful constants: $e = 1.6 \times 10^{-19}$ C; $\varepsilon_o = 8.85 \times 10^{-12}$ C^2/Nm^2 ; $\mu_o = 4\pi \times 10^{-7}$ N/A^2

15 pts 1) Consider two grounded conducting sheets, one lying in the *x*-*y* plane, the other parallel to it and distance *a* above it. The sheets extend indefinitely in the *z*-direction, and exist for $x > 0$. The potential along a strip at $x = 0$, $0 < y < a$, and all *z* is given by:

$$V(0, y) = V_o \sin^3(\pi y/a).$$

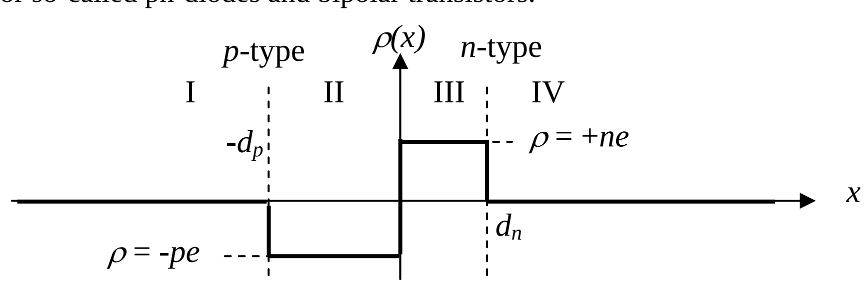
Use separation of variables to solve Laplace's equation and find the potential in the gap for $x > 0$.



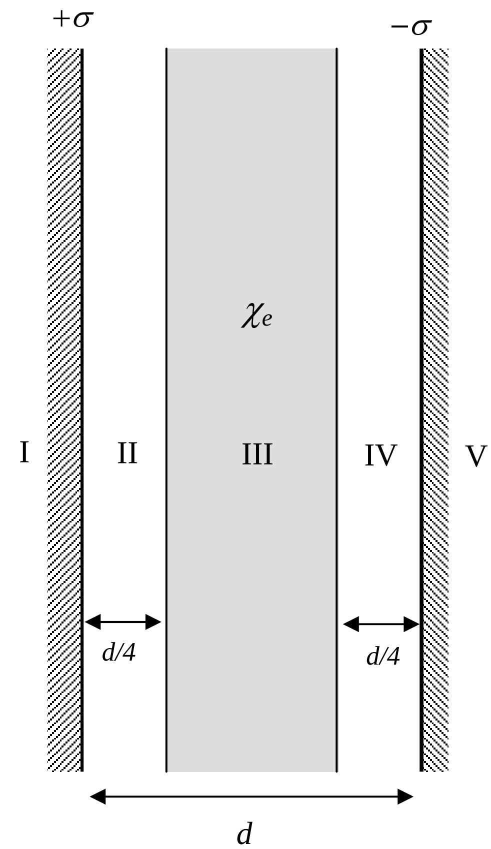
20 pts 2) Near the interface between a piece of n-type and a piece of p-type semiconductor there is a region where charged impurities form a uniform layer of volume charge density of $\rho = ne$ for a thickness d_n on the n-type side and a layer with $\rho = -pe$ appears for a thickness d_p on the p-type side. Here *n* is the density (i.e. number/unit volume) of n-type impurities and *p* is the density of p-type impurities. Let $p = n/2$. The charge density is then only a function of *x* as shown in the graph below. A candidate potential is (with $V_\infty, V_{-\infty}, C_p, C_n$ constants):

$$V(x) = \begin{cases} V_{-\infty} \text{ for } x < -d_p \\ \frac{pe}{2\varepsilon_o}(x + d_p)^2 + C_p \text{ for } -d_p < x < 0 \\ -\frac{ne}{2\varepsilon_o}(x - d_n)^2 + C_n \text{ for } 0 < x < d_n \\ V_{+\infty} \text{ for } d_n < x \end{cases}$$

a) What value of the ratio d_p/d_n ensures that the entire system has no net charge?
b) Verify that the above expressions for $V(x)$ are correct in the region where they apply (that is, verify that it is consistent with the charge density in its region).
c) Find an expression for the electric field in each region: I, II, III, IV.
d) Use continuity of the potential at $x = -d_p$ and $x = +d_n$ to find the values of C_n and C_p.
e) Use continuity of the potential at $x = 0$ to find an expression for the potential difference $\Delta V = V_\infty - V_{-\infty}$. This is the potential difference one would measure if one placed a voltmeter across the n-side to the p-side of such a junction. This effect is the basis for so-called pn-diodes and bipolar transistors.

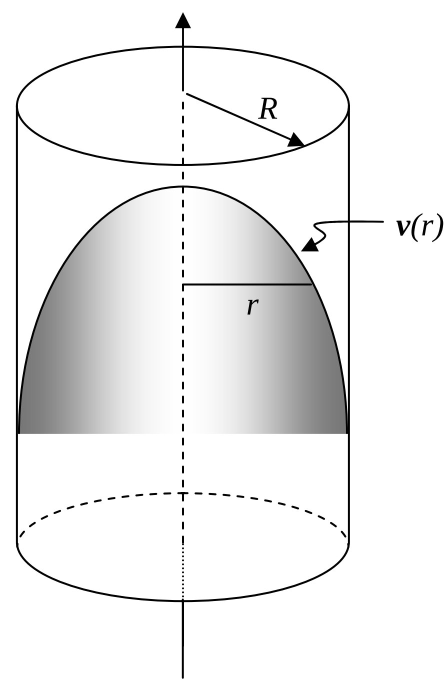


15 pts 3) Two infinite planar conductors have free surface charge densities $+\sigma$ and $-\sigma$, respectively. The planes are parallel and separated by a distance *d*. The volume between the planes contains an infinite slab of dielectric with susceptibility χ_e and thickness *d*/2. The slab is parallel to the conductors and its surfaces are a distance *d*/4 from the nearest conductor as shown in the figure. Space is therefore divided into five regions, I, II, III, IV, and V.



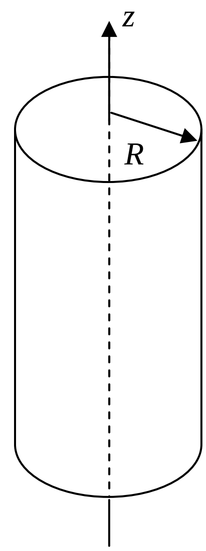
a) What is ***E*** in each region of space?
b) What is ***D*** in each region?
c) Find an expression for the bound surface charge density on each surface, and the volume charge density in, the dielectric.
d) What is the potential difference between the conductors?
e) Recall that the capacitance of a configuration of conductors is C = Q/V. What is the capacitance per unit area of this arrangement? What would it be if the gap between the conductors were empty? Which situation enables one to store more charge for a given potential difference?

20 pts 4) A long plastic pipe carries a fluid with a uniform volume charge density ρ_o. The velocity of the fluid is a function of its distance from the axis and is given by $\vec{v} = v_o(1 - \frac{r^2}{R^2})\hat{z}$ as shown in the sketch.



a) What is the current density ***J*** and the total current *I* carried by this pipe.
b) Find an expression for the magnetic field for *r* < *R* and for *r* > *R*.
c) Is there any property of the ***B*** outside that gives you an indication of the functional form of the current density inside the pipe? What is the most you can say?

15 pts 5) Suppose that a long cylindrical rod of radius R is made of a magnetic material with magnetization $\vec{M} = kr^3\hat{z}$.



a) Find the surface and bulk bound current densities.
b) Determine the magnetic field inside and outside the rod (note: you may wish to consider the rod as a series of concentric spherical shells and add [i.e. integrate] the contribution from each shell to find the field).
c) Does the magnetic field satisfy the anticipated boundary condition at $r = R$? Justify your answer by calculation.

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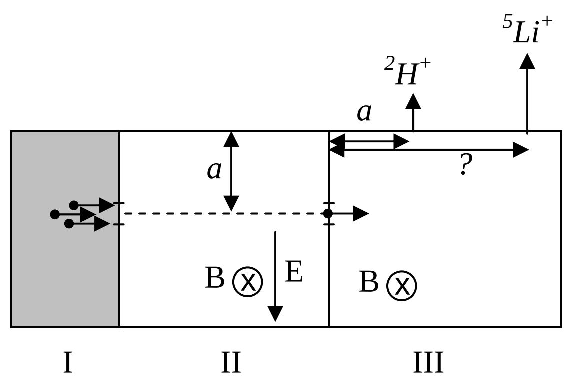
15 pts 6) A plasma chamber (I) produces a collection of energetic, singly ionized (*i.e.*, $q = +e$) particles of ^{2}H and ^{5}Li. Some of the particles moving parallel to the *x*-axis pass through a narrow slit, that is *a* distance a from the top of the chamber, and travel trough a region (II) where there is both a uniform electric field $-E_o\hat{y}$ and a uniform magnetic field $-B_o\hat{z}$. Some of these go through the region undeflected and pass through a second slit at the same height, entering region III where there is no electric field, only the magnetic field $-B_o\hat{z}$. The ^{2}H$^+$ ions are observed to exit the top of the second chamber a distance *a* from the entrance wall.



a) What is the velocity of the particles that pass through region II undeflected?
b) How far from the entrance wall will the ^{5}Li$^+$ ions emerge from the second chamber?
c) In terms of the E_o and B_o, what fields must be used to get the same Li ions to exit the chamber where the H ions originally exited?