

Name: _____

Ground rules:

- Open Book
- Closed notes
- You may consult one page of handwritten notes
- A calculator is allowed but will not be useful
- Write your answer directly on these sheets (continue onto back, if necessary)

There are four questions of between 20-30 points each. Pace yourself accordingly.

If you know the formulas for “standard cases” (e.g., the electric field or potential a certain distance from a point or line or plane charge), you may use these results without derivation unless the problem specifically asks you to derive it.

Partial credit will be given. Do as many parts of a problem as possible. If you are stuck on the first part of a problem, you should still try to say something about the later parts if possible.

Be sure to check that you have done all parts of all questions.

Feel free to raise your hand to ask a question.

Good luck!!

Problem 1 (25 points)

A sphere carries a spherically symmetric interior charge density

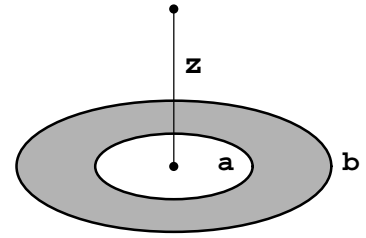
$$\rho(r) = \rho_0 \left(1 - \frac{r}{R}\right)$$

for $r < R$, and there is vacuum for $r > R$. Using Gauss's Law, find the electric field both *inside* and *outside* the sphere.

Problem 2 (25 points)

Consider an annulus (flat circular ring) with inner radius a and outer radius b , carrying uniform surface density σ .

- Find the total charge of the annulus.
- Find the electric potential at a point on the axis located a distance z from the center of the hole.
- How much work is required to bring a charge Q from infinity and place it at the center of the hole?



Hints: $\int \frac{t dt}{\sqrt{c^2 + t^2}} = \sqrt{c^2 + t^2}$. Also note that when $c \gg d$, $\sqrt{c^2 + d^2} \simeq c + d^2/2c$.

Problem 3 (20 points)

For a system with *spherical* symmetry, the electric field is known to be $\mathbf{E} = Ar^2 \hat{r}$, where A is a constant.

- a) Find the electric potential $V(r)$, using $r = 0$ for your reference point \mathcal{O} .
- b) Find the charge density $\rho(r)$.

Hint: The two parts are intended to be independent; you should not need the answer to (a) in order to get (b).

Problem 4 (30 points)

A uniform volume charge ρ_0 fills the space between the planes $x = -R$ and $x = R$, except that there is an empty cylindrical cavity of radius R extending along the z -axis. The figure shows a cross-sectional view. (Beware: the circle in the figure shows a *cylinder*, not a *sphere*.)

- Find the electric field \mathbf{E} at the point A .
- Find the electric field \mathbf{E} at the point B whose coordinates are $(R, R, 0)$.

Hint: Consider the charge distribution to be the superposition of a positively charged slab and a negatively charged cylinder!

