Final Exam Info

- **Wed. Dec 20th 8-11am room SEC 118**
  - Cumulative, but with a focus on the material since the midterm
  - it will be **closed book** but you are allowed to bring **TWO** equation sheets (max)
    - two 8.5" x 11" sheet of paper with formulas and notes to consult during the exam. You may write on both sides of these cheat sheets.
    - you should also bring a couple of pencils and a scientific calculator.
  - check website for more information.

- **Wednesday’s class will be a final exam review session – you are strongly encouraged to attend!** I will post slides in advance.

- **I will hold office hours Tuesdays, December 12 and 19 from 3:30-4:30pm.**
How to study

- Reed chapters 1-10
  - Review the examples in the book!
- Review Eisberg&Resnick material on reserve at MSLC
- Review all lectures
- Review all homework problems
- Go over Wednesday’s review session material
- Prepare your formula sheets
- Form study groups
Numerical Solutions to SE

- Whether the solution to SE is exact or approximate, often the algebra is extremely tedious.

- It is possible to solve SE for complicated potentials with numerical integration.

- A drawback to this technique is that the eigenstates need to be evaluated one at a time.
If we express:

\[ \begin{bmatrix} m \\ l \end{bmatrix} = \hat{A} \]

\[ [E] = eV \]

SE becomes:

\[ \frac{d^2 \Psi(x)}{dx^2} = -0.26246 m [E-V(x)] \Psi(x) \]
Numerical Integration

- Divide domain \( x \) into a number of equally spaced discrete points separated by step size \( \Delta x \)
- Start to add small increments to \( \Psi(x) \), stating at point \( x_0 \), based on derivatives multiplied by step sizes.

1) Need SE

\[
\frac{d^2 \Psi(x)}{dx^2} = -0.26246 m \left[ E - V(x) \right] \Psi(x)
\]

2) Need second derivative

\[
\left( \frac{d^2 \Psi}{dx^2} \right)_{x_0} = \left[ \left( \frac{d\Psi}{dx} \right)_{x_0 + \Delta x} - \left( \frac{d\Psi}{dx} \right)_{x_0} \right] / \Delta x
\]

\[
\left( \frac{d\Psi}{dx} \right)_{x_0 + \Delta x} = \left( \frac{d^2 \Psi}{dx^2} \right)_{x_0} \Delta x + \left( \frac{d\Psi}{dx} \right)_{x_0}
\]

3) Need Taylor series expansion

\[
\Psi(x_0 + \Delta x) = \Psi(x_0) + \left( \frac{d\Psi}{dx} \right)_{x_0} \Delta x + \left( \frac{d^2 \Psi}{dx^2} \right)_{x_0} \frac{\Delta x^2}{2} + \ldots
\]
Numerical Integration

1. Specify: \( V(x) \), \( \Delta x \), \( x_0 \), values of \( \Psi \) at \( x_0 \)
2. Make a trial guess at energy eigenvalue \( E \)
3. Begin integration “cycle” using SE and computing \( (d^2\Psi/dx^2) \) at \( x_0 \)
4. Use above to compute \( \Psi(x_0 + \Delta x) \) using Taylor series expansion
5. Use above results to compute \( (d\Psi/dx) \) at \( x_0 + \Delta x \)
6. Go back to step 3 and repeat, until \( x \) reaches an appropriate upper limit.
Example

Let’s use numerical integration to evaluate the ground state energy solution for the linear potential we’ve considered before

\[ V(x) = \begin{cases} 
\infty & x < 0 \\
\alpha x & x \geq 0 
\end{cases} \]

\[ \alpha = 1 \text{eV/Angstrom} \]
Energy eigenvalues for quantum state $n$ we found earlier can be written as:

$$\epsilon_n = \eta_n \alpha^{2/3} \varepsilon^{1/3}$$

- $\eta_n$ is a dimensionless numerical coefficient

$$\alpha^{2/3} \varepsilon^{1/3} = 1.5619 \text{eV}$$

$$E_n / \eta_n = 1.5619 \text{eV}$$

- From table, a good guess for ground state $\eta$ is $\sim 1.8$, which makes $E \sim 2.8 \text{eV}$

- So a good first trial guess for $E$ is $\sim 3 \text{eV}$

<table>
<thead>
<tr>
<th>Method</th>
<th>$\eta_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uncertainty Principle</td>
<td>$&gt; 1.191$</td>
</tr>
<tr>
<td>WKB Approximation</td>
<td>2.811</td>
</tr>
<tr>
<td>Variational</td>
<td>$&lt; 2.476$</td>
</tr>
<tr>
<td>Improved Variational</td>
<td>$1.560 - 2.476$</td>
</tr>
<tr>
<td>Exact</td>
<td>2.338107</td>
</tr>
</tbody>
</table>
- Graph diverges at $x \sim 6$ Angstroms

- If we were to increase guess of $E$ to 3.1eV, divergence happens at a slightly higher $x$

- At $E$ between 3.6 and 3.7eV, go from positive to negative divergence (more precisely at around $3.6510eV$), so:

  \[ \eta_1 \sim \frac{3.6510}{1.5619} = 2.3375 \]

  About 0.24% lower than exact value
Interpretations of QM: References

- Many text books, e.g.:
  - Griffiths: “Introduction to Quantum Mechanics”, Pearson, 2005
Copenhagen Interpretation

- (Bohr, Heisenberg)
- A wave function $\Psi$ provides a complete characterization of the state of a system.
- In general $\Psi$ is a superposition of eigenstates:
  \[ \Psi = \sum a_n \Psi_n \]
- Measurement yields an eigenvalue, and $\Psi$ abruptly collapses to $\Psi_n$.
- Cannot predict which $\Psi_n$
  - Probability is $a_n^* a_n$
- Ohanian: Measurements produce unpredictable, discontinuous changes in $\Psi$, which do not obey S.E., when the collapse occurs.
Hidden Variables Interpretation

- \( \Psi \) is not a complete description.

- There are hidden variables and if we knew them, we’d avoid probabilities and predict exactly how the system will evolve.
The Paradox of Shrodinger’s Cat

- A cage contains a cat, some radioactive atoms and a geiger counter.
- There is a 50% chance that in the next hour an atom will decay, counter will click, which releases a hammer that breaks the bottle of poison and kills the cat.
- In QM how do we describe the state of the cat after an hour?
  \[ \Psi_{cat} = \frac{1}{\sqrt{2}} \Psi_{alive} + \frac{1}{\sqrt{2}} \Psi_{dead} \]
- But we never see a superposition. When we look, \( \Psi_{cat} \) collapses and cat is either alive or dead.
- But when and how did the collapse occur?
Interpretations

- **Idealistic or subjective interpretation of Von Neumann and Wigner:**
  - Collapse occurs when observer’s consciousness registers that the cat is alive or is dead!
- **Most people disagree.**
  - e.g. Ohanian: What level of consciousness is sufficient to bring about collapse? Is human consciousness required, or is that of a cat or a mosquito sufficient?
Interpretations

- **Many-worlds interpretation** of Everett:
  - No collapse.
  - So if we make an observation and obtain a live cat, that causes the universe to split into a parallel branch in which the cat is dead!
  - So the universe is continually splitting into myriad branches which remain forever unaware of each other.
- Main objection: this is not testable!
Interpretations

- **Realism interpretation:**
  - At any instant, cat is either alive or is dead.
  - Physical reality exists whether or not an observation is made.
  - Einstein: Is the moon not there if nobody is looking?
Phenomenolism

- Copenhagen interpretation adopts ideas of logical positivism or phenomenalism.
- The only meaningful statements we can make about a physical system are those that are testable and measurable.
- So $\Psi_{\text{cat}}$ does not imply that cat is a superposition of life and death.
- What is meaningful is that it predicts the probability of the cat being alive or dead WHEN an observation is made.
Ohanian: We must not regard $\Psi$ as some kind of snapshot of the instantaneous configuration of the system.

Goswami:

- Heisenberg said that $\Psi$ represents the not real system but our knowledge of the system.
- The collapse of the wave function is not a real physical event, but represents a change in our knowledge of the system as a result of our measurement.

A related concept is inseparability:

- Quantum systems cannot be separated from the measurement process.
Bohm’s Hidden Variables Alternative

- Particles do possess definite positions and velocities but these features are hidden from view.

- Wave function of a particle interacts with the particle itself - it guides or pushes the particle around - in a way that determines its subsequent motion.

- Changes to the wave function in one location are able to immediately push a particle at a distant location.

- Hence explicitly non-local.

- Main criticism:

  - It is a central element of Bohm’s theory that the wave function can exert faster-than-light influences on the particle it pushes.
Conclusions

- Majority of physicists accept Copenhagen interpretation and the acronym
  - SUAC: Shut up and calculate
  - In other words, regard QM as a calculational tool.
Summary/Announcements

- Next time: REVIEW FOR FINAL EXAM

- I will hold final office hours this week and next week on Tuesdays from 3:30-4:30pm.

- Please fill out the course survey at:
  http://ctaar.rutgers.edu/sirs/current-surveys

- Final exam: Wed. Dec 20th 8-11am room SEC 118
  - More details on earlier slides and this Wed during review

- Now time for a QUIZ …