Quantum Mechanics and Atomic Physics

Monday & Wednesday 1:40-3:00 SEC 208

http://www.physics.rutgers.edu/ugrad/361

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Physics 361
Quantum Mechanics and Atomic Physics
Fall 2018

Mondays and Wednesdays, 1:40-3:00 PM, SEC-208, Busch Campus

Prerequisites: 01:640:CALC4; 01:750:228 or 273 or permission of instructor.

Catalog description of this course: Introductory quantum mechanics: matter waves, uncertainty principle, stationary states and operators; the Schrodinger equation and its solutions for simple potentials; the hydrogen atom, quantization of angular momentum, spin; complex atoms and molecules.

Primary Text book: "Quantum Mechanics" by Bruce Cameron Reed (Jones and Bartlett Publishers. Copyright 2008)
Additional reading material: I will use selected material from "Quantum Physics of Atoms, Molecules, Solids, Nuclei, and Particles" by R. Eisberg and R. Resnick (John Wiley and Sons, 2nd Edition). I will mainly only refer to Chapters 8-10 from this secondary textbook.
Both of these textbooks are on reserve at the MSLC on Busch Campus (please only use them at the MSLC and do not take them home with you!).

Course Policies and Student Resources: Please consult this link for more information. Also, please consult me as early as possible if you have a disability that might interfere with an optimal learning experience.

Course Links:
- Grading
- Syllabus
- Lecture notes
- Homework
- Quizzes
- Exams
- Online Gradebook (coming soon)
- Announcements (latest entry: )

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Office hours: Tuesdays 3:30-4:30pm
Textbook(s)

- "Quantum Mechanics" by Bruce Cameron Reed (Jones and Bartlett Publishers, Copyright 2008)
  - I will primarily follow the material in this book
  - I encourage you to read the material prior to attending class

- Additional reading material:
  Later in the semester I will use selected material from "Quantum Physics of Atoms, Molecules, Solids, Nuclei, and Particles" by R. Eisberg and R. Resnick (John Wiley and Sons, 2nd Edition)
  - I will only cover the selected material (Chapters 8-10) from this secondary textbook.

- Both of these textbooks are on reserve in MSLC on Busch Campus. Please only use them at the MSLC and do not take them home with you!
Homework

- The homework will also primarily come from the book
  - You will have weekly homework assignments due at the beginning of class on Mondays
  - First homework is due on September 17
Grades

- Your grade will be based on four components:
  - weekly homework (25%)
  - frequent in-class quizzes (20%)
  - a midterm exam (20%)
  - a final exam (35%)
- Attendance is strongly advised.

Late homeworks will not be accepted.
- 2 lowest homework grades will be dropped

There are no make-ups for missed quizzes.
- Lowest quiz grade will be dropped
Introduction

- Qualitatively, Quantum Mechanics can be viewed as a theory that correctly describes the behavior of microscopic material particles but incorporates the predictions of Newtonian mechanics on macroscopic scales.

- We will first review some of the facts that led physicists in the early 20th century to the realization that Newtonian mechanics is invalid in the realm of atomic scale phenomena.
Kirchhoff’s Laws of Spectroscopy

- In the 1850’s, Gustav Kirchhoff observed:
  - **Light from hot object through prism** ⇒ **continuous spectrum**
  - **Light through cool gas** ⇒ **same spectrum but with certain wavelengths of light removed**
  - **Light emitted by hot gas alone** ⇒ **series of bright lines at certain wavelengths**
Thermal Radiation

1. Objects can **reflect** light
2. Objects can **emit** light
   a) Thermal (continuous) spectrum: all wavelengths
   b) Line spectrum: discrete wavelengths

- Explaining discrete line spectra posed a challenge to builders of atomic models.
- Continuous spectra also posed challenges to classical physics.
  - All materials emit thermal radiation or “blackbody” radiation
  - Hotter temperatures yield shorter wavelengths
  - Attempts to find a mathematical description of thermal radiation based on classical models failed.
There are three main laws that characterize thermal radiation:

1. **Wien's displacement law**
2. **Stefan-Boltzmann law**
3. **Planck’s law of radiation**
Wien’s displacement law

- A “blackbody” absorbs all light shining on it
  - No reflection
  - But it emits thermal radiation
- Wien’s Law (In 1893, a general form for blackbody radiation):
  - The main frequency (or color) of the emitted radiation increases as the temperature increases.
    \[ \lambda_{\text{max}} T = 2.8979 \times 10^{-3} \, m \cdot K \]
    \[ \nu_{\text{max}} / T = 5.88 \times 10^{10} \, \frac{Hz}{K} \]
  - As T increases:
    \( \lambda_{\text{max}} \) decreases / \( \nu_{\text{max}} \) increases
    and the peaks are displaced.

\( T_1 < T_2 < T_3 \)
Stefan-Boltzmann law

- In 1879, J. Stefan found experimentally that the total radiancy of a hot solid was proportional to $T^4$.
- Integrate to get the Total Radiancy:
  \[ R_{tot} = \int R(\nu)d\nu = \int R(\lambda)d\lambda \]
- The total amount of radiation, of all frequencies, goes up very very fast as the temperature rises.
- Power per unit area:
  (we saw this from the units on previous page)
  \[ R_{tot} = \text{Power} / \text{Area} = \sigma \cdot T^4 \]
  \[ \sigma = 5.67 \times 10^{-8} \text{W} / \text{m}^2 \cdot \text{K}^4 \]
  (Stefan - Boltzmann constant)
Rayleigh-Jeans Theory and the Ultraviolet Catastrophe

Rayleigh-Jeans used classical physics to derive the “energy density” $\rho(\nu)$ and $\rho(\lambda)$.

Cavity radiation $\approx$ Blackbody radiation

$$R(\nu) = \frac{c}{4} \rho(\nu), \quad R(\lambda) = \frac{c}{4} \rho(\lambda)$$

$$\rho(\nu) = \frac{8\pi\nu^2 kT}{c^3}$$

$$\rho(\lambda) = \frac{8\pi kT}{\lambda^4}$$

“Ultraviolet Catastrophe”

$$\lim_{\nu \to \infty} \rho(\nu) = \infty$$

Classical theory prediction of blackbody radiation seriously discrepant with experimental data!
Max Planck’s hypothesis (1900):

- Energy of light at a given $\nu$ is quantized:

$$E = nh\nu, \quad n = 1, 2, 3, \ldots$$

- $h = \text{Planck's constant} = 6.63 \times 10^{-34} \text{ J} \cdot \text{s}$

- Thermal radiation, even at a single temperature, occurs at a wide range of frequencies. How much of each frequency is given by Planck's law of radiation.

- Energy density (energy/unit frequency or energy/unit wavelength):

$$\rho(\nu) = \frac{8\pi\nu^2}{c^3} \frac{h\nu}{e^{h\nu/kT} - 1}$$

$$\rho(\lambda) = \frac{8\pihc}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$$

This ushered in QM!
Example

- Show that the spectral energy density is proportional to $T$ in the low-frequency or so-called classical region.

\[
\rho(\nu) = \frac{8\pi \gamma^2}{c^3} \frac{h\nu}{e^{h\nu/kT} - 1}
\]

\[
\frac{1}{e^{h\nu/kT} - 1} \approx \frac{kT}{h\nu} \quad \text{for} \quad \frac{h\nu}{kT} \ll 1
\]

\[
\rho(\nu) \approx \frac{8\pi h\nu^3}{c^3} \cdot \frac{kT}{h\nu} = \frac{8\pi \gamma^2}{c^3} kT
\]

$\rho(\nu) \propto T$

Rayleigh-Jeans result!
Example

Wien’s displacement laws for frequency and wavelength are:

\[ \lambda_{\text{max}} T = 2.8979 \times 10^{-3} \text{ m} \cdot K = 0.2014 \frac{hc}{k} \]

\[ \nu_{\text{max}} / T = 5.88 \times 10^{10} \frac{Hz}{K} = 2.8214 \frac{kT}{h} \]

a) Show that these are consistent with Planck’s expressions for the energy densities, \( \rho(\nu) \), \( \rho(\lambda) \)

b) From the two forms of Wien’s Law, why does the product \( \nu_{\text{max}} \lambda_{\text{max}} \) not equal c?!
To get $v_{\text{max}}$:

\[
\frac{dP}{dv} = 0 \quad \text{where} \quad P(v) = \frac{8\pi v^2}{c^3} \frac{h v}{e^{hv/kT} - 1}
\]

\[
\frac{dP}{dv} = 0 = \frac{8\pi h}{c^3 (e^{hv/kT} - 1)^2} \left[ (e^{hv/kT} - 1) 3v^2 - v^3 e^{hv/kT} \right]
\]

\[
\Rightarrow 3v^2 e^{hv/kT} - 3v^2 - \frac{h v^3 e^{hv/kT}}{kT} = 0
\]

\[
\Rightarrow 3 - 3e^{-hv/kT} - \frac{h}{kT} v = 0
\]

This is a transcendental eqn. But if you substitute $v = 2.8214 \frac{kT}{h}$, the equation becomes $3 - 0.81786 - 2.8214 = 3 - 0.81786 - 2.8214$ which is indeed zero!
To get $\lambda_{\text{max}}$:

$$\rho(\lambda) = \frac{8\pi \hbar c}{\lambda^5} \frac{1}{e^{\hbar c/\lambda kT} - 1}$$

$$\frac{d\rho}{d\lambda} = \frac{8\pi \hbar c}{(e^{\hbar c/\lambda kT} - 1)^2} \left[ (e^{\hbar c/\lambda kT} - 1) (-5\lambda^{-6}) - 7\lambda^5 \left( \frac{\hbar c/\lambda kT}{\left(\frac{\hbar c}{\lambda kT}\right)^2} \right) \right]$$

Simplify by multiplying through by $\lambda^2 e^{-\hbar c/\lambda kT}$.

$$\Rightarrow \frac{\hbar c}{kT} + 5\lambda e^{-\hbar c/\lambda kT} = 0$$

$$-5\lambda = 0$$

Also a transcendental eqn. Substitute $\lambda = 0.2014 \frac{\hbar c}{kT}$ to get $\frac{\hbar c}{kT} + 0.0070 \frac{\hbar c}{kT} - 1.0070 \frac{\hbar c}{kT} = 0$.
b) $V_{\text{max}} = \lambda_{\text{max}} = \left(2.8214 \frac{kT}{h}\right) \left(0.2014 \frac{hc}{kT}\right)$

$= 0.5682 \frac{c}{\nu}$

It is not true that $\rho(v) = \rho(\lambda)$.

What is true is that

$\int |\rho(v) dv| = \int |\rho(\lambda) d\lambda|$

$\rho(v) = \rho(\lambda) \left| \frac{d\lambda}{dv} \right| = \rho(\lambda) \frac{c}{\nu^2}$ since $\lambda = \frac{c}{\nu}$. 
Planck’s hypothesis remained dormant until 1905 when Einstein adapted it to explain the photoelectric effect.

- Light shines on metal plate and dislodges electrons which travel to cathode plate.
- Circuit has photocurrent $i$. Apply stopping potential. Stopping potential $V_0$ is needed to kill photocurrent.
Photoelectric Effect

\[ eV_0 = \frac{1}{2} m v_{\text{max}}^2 \]

- But other features of the photoelectric effect made no sense ....
Photoelectric Effect

- $V_0$ vs. $\nu$:
  - Why are the lines parallel?
  - For each substance (Cu, K, Cs) why is there a threshold frequency below which no photoemission occurs?
Einstein’s Explanation (1905)

- Light consists of individual particles (“photons”) each of energy:
  \[ E = h\nu = \frac{hc}{\lambda} \]

- Photoelectric effect: each photon liberates one electron: one-on-one process.

- Intensity of light \( I = Nh\nu \) (in \( W/m^2 \)) and \( N = \# \) photons/area/second

- Energy conservation:
  \[ h\nu = \omega_0 + \frac{1}{2}mv_{\text{max}}^2 = \omega_0 + eV_0 \]

- “work function” \( \omega_0 = \) binding energy of the least tightly bound electrons
Einstein’s explanation

- These plots are of $eV_0 = h\nu - \omega_0$

- All lines have same slope $h$! But $\omega_0$ depends on the specific metal and $\omega_0 = h\nu_{\text{threshold}}$
Need a bound electron: Proof 1

- To prove: photoelectric effect requires a bound electron. It can’t be free.

P conservation: $p_\gamma = p_e$ so $E_\gamma = p_\gamma c = p_e c$

E conservation: $E_\gamma + m_e c^2 = \sqrt{(p_e c)^2 + (m_e c^2)^2}$

Since $E_\gamma = p_e c$,

$(p_e c + m_e c^2)^2 = (p_e c)^2 + (m_e c^2)^2$

$\Rightarrow (p_e c)^2 + 2 p_e m_e c^3 + (m_e c^2)^2 = (p_e c)^2 + (m_e c^2)^2$

$\Rightarrow p_e = 0!$ Hence impossible.
Need a bound electron: Proof 2

- More elegant proof: consider center of mass frame

Total $\mathbf{p}_{\text{before}} = 0$ by definition of C.M.

$\Rightarrow$ Electron is at rest after the collision.

But now energy is manifestly not conserved!

\[
\begin{align*}
E_{\text{before}} &= E'_\gamma + m_ec^2 + K'_e \\
E_{\text{after}} &= m_ec^2
\end{align*}
\]

Can’t be equal!
Summary

- The need to introduce the hypothesis that some physical quantities are quantized grew out of the inability of classical physics to provide adequate understanding of phenomena such as thermal radiation.
  - Planck’s blackbody radiation formula
  - Explained phenomena such as blackbody radiation and the photoelectric effect.
  - Light regarded as stream of particles, photons.

- Next time --
  - The Rutherford-Bohr atom
  - de Broglie Matter-Waves

- First homework due on Monday Sept 17! Assignments will show up on the course webpage.