Solution to HW#11

1. Reed Chapter 8

Problem 8-8

Classify the following neutral atoms as to whether they will be fermions or bosons: $^{29}\text{Si}$, $^{40}\text{Ca}$, $^{69}\text{Ga}$, $^{105}\text{Pd}$, $^{137}\text{Cs}$, $^{235}\text{U}$.

For a neutral species of the form $^A\!\!\!\underline{X}$ the total number of protons, neutrons, and electrons is $A+Z$. For the cases given these are 43, 60, 100, 151, 192 and 327. These constituents are all spin-1/2; if the total is even we have a boson and if it is odd we have a fermion. Hence we have F,B,B,F,B,F.

2. Reed Chapter 8

Problem 8-9

In Problem 4-16 it was remarked that early in the history of nuclear physics the electrically neutral mass of nuclei now attributed to neutrons was considered to arise from neutral particles composed of combinations of protons and electrons (as opposed neutrons in their own right as fundamental particles). That problem explored the implications of the uncertainty principle for that theory. Here we have a look at the spin statistics of the situation. Consider a nitrogen-14 ($^{14}\!\!\!\underline{N}$) nucleus. If the “protons + electrons” model were correct, would you predict N-14 to be a spin-1/2 or spin-1 system? What about in the case of the “protons + neutrons” model? Spectroscopic evidence indicates that N-14 is a spin-1 system: which model does this support?

In the “protons + electrons” model the total number of particles is 21: 14 protons plus 7 electrons, for a net positive charge of 7. This would be a spin-1/2 system. In the “protons + neutrons” model the number of constituents is 14, hence a spin-1 system. The spectroscopic evidence favors the neutrons + protons model, as does the uncertainty principle argument.
3. Sodium, Na, has 11 electrons.

We did this in class!
This is “case I” where there is only one electron in the outer shell.
4. Let’s use Hund’s rules:
  Lowest energy for maximum $S'$. 
  For each $S'$, lowest energy for maximum $L'$. For each multiplet, lowest energy for minimum $J'$. 

\[
\begin{align*}
S' &= |s_1 - s_2|, \quad (s_1 + s_2) = 0, 1 \\
L' &= |l_1 - l_2|, \quad (l_1 + l_2) = 2, 3, 4 \\
S' &= 1, L' = 4 \\
J' &= |L' - S'|, \quad (L' + S') = 3, 4, 5
\end{align*}
\]

So, the lowest energy is for $J'=3$:

\[
\Rightarrow S' = 1, \ L' = 4, \ J' = 3 \Rightarrow 3 \, G_3
\]

\[2S' + 1 = 3 \quad L' = 4 \Rightarrow G\]
The Landé interval rule gives:

\[
\frac{3\text{D}_3 - 3\text{D}_2}{3\text{D}_2 - 3\text{D}_1} = \frac{2+1}{1+1} = \frac{3}{2}
\]

Note that the \(3\text{D}_1\) and \(3\text{D}_2\) states went below the line marked \(S'=1\) because the fine structure energy splitting goes as:

\[
\Delta E_{FS} \propto J'(J' + 1) - L'(L' + 1) - S'(S' + 1)
\]
6. We are considering the 2p3s configuration of Carbon. This is an orbital excitation of one electron in Carbon to the 3s state.

\[ l_1 = 1, \quad s_1 = \frac{1}{2} \Rightarrow s' = 0, 1 \]

\[ l_2 = 0, \quad s_2 = \frac{1}{2} \Rightarrow l' = 1 \]

\[ s' = 0, \quad l' = 1 \Rightarrow J' = 1 \]

\[ s' = 1, \quad l' = 1 \Rightarrow J' = 0, 1, 2 \]

\[ ^3P_0, ^3P_1, ^3P_2 \]

The Landé interval rule gives:

\[ \frac{3P_2 - 3P_1}{3P_1 - 3P_0} = \frac{1+1}{0+1} = \frac{2}{1} = 2 \]

Again, like before, note that the \(^3P_0\) and \(^3P_1\) states went below the line marked \(S' = 1\) because the fine structure energy splitting goes as:

\[ \Delta E_{FS} \propto J'(J'+1) - l'(l'+1) - s'(s'+1) \]
7. Let’s calculate the Landé g-factors:

\[ g = 1 + \frac{J'(J'+1) + S'(S'+1) - L'(L'+1)}{2J'(J'+1)} \]

\[ = 1 + \frac{(1)(2) + (0)(1) - (1)(2)}{2(1)(2)} \]

\[ = 1 \]

This should be obvious since \( g=1 \) whenever \( S'=0 \). Also this state splits into \( 2J'+1=3 \) levels.

\[ 3P_2: \quad g = 1 + \frac{(2)(3) + (1)(2) - (1)(2)}{2(2)(3)} = \frac{3}{2} \]

This state splits into \( 2J'+1=5 \) levels.

\[ 3P_1: \quad g = 1 + \frac{(1)(2) + (1)(2) - (1)(2)}{(2)(1)(2)} = \frac{3}{2} \]

This state splits into \( 2J'+1=3 \) levels.

\[ 3P_0: \quad g = 1 + \frac{(0)(1) + (1)(2) - (1)(2)}{2(0)(1)} = 1 + \frac{0}{0} \]

The 0/0 doesn’t really matter. Since \( J'=0 \) for \( 3P_0 \), the state doesn’t split and has no magnetic moment, so \( g \) isn’t really meaningful.
The splitting of these levels in a weak external magnetic field is:

\[
\Delta E = g m_J \frac{\mu_B B}{\mu_B B}
\]

\[
\Delta E = g m_J \frac{\mu_B B}{\mu_B B}
\]

\[
\Delta E = g m_J \frac{\mu_B B}{\mu_B B}
\]
Note that the spacing between substates in $^3P_1$ and $^3P_2$ is the same because the g-factor is the same. Spacing in $^1P_1$ is less because the g-factor is less.

Now, why did we get the same g-factor for $^3P_1$ and $^3P_2$?

Well, note that $S'=1$ and $L'=1$ for both. So,

$$g = 1 + \frac{J'(J'+1) + S'(S'+1) - L'(L'+1)}{2J'(J'+1)}$$

Which is independent of $J'$, as long as $J' \neq 0$. 