Systems with $N \neq \text{const}$

Gibbs Statistics

Reservoir $U_R$, $N_R$ $\leftrightarrow$ System $E$, $\mu$
\[
\frac{p(s_1)}{p(s_2)} = \frac{R_e(s_1)}{R_e(s_2)} e^{\frac{S_R(s_1)}{k}} = e^{\frac{S_R(s_2)}{k}} = e^{dS_R/k}
\]

\[
\to \quad S_R = \frac{1}{T} \left( dU_R + P dV_R - \mu \cdot dN_R \right)
\]
\[
\frac{p(s_1)}{e^{-\frac{E(s_1) - \mu w(s_1)}{kT}}} = \frac{p(s_2)}{e^{-\frac{E(s_2) - \mu w(s_2)}{kT}}} = \text{const}
\]
\[ p(s) = \frac{1}{\tilde{Z}} \ e^{-\frac{E(s) - \mu N(s)}{kT}} \]

\[ \tilde{Z} = \sum e^{-\frac{E(s) - \mu N(s)}{kT}} \]

\( \tilde{Z} \) - grand partition function

- Gibbs factor
Quantum Statistics (Chapter 7)

will consider non-interacting particles
Recall, not too dense systems:

\[ \Omega = \mathcal{Z} \quad (\text{distinguishable particles}) \]

\[ \Omega = \frac{1}{N!} \mathcal{Z} \quad (\text{in distinguishable particles}) \]

works if no more than one particle occupies each state.
Now will consider "dense" systems

Example 2 particles,
5 energy states, \( E = 0 \)
\( \text{so } \hat{H} = \hat{E} \)

Show states as \( |00000\rangle \)
Distinguishable particles:

Particle A: 5 states
Particle B: 5 states

\[ N = 5 \times 5 = 25 \]
\[ z = 25 \]
Indistinguishable particles

\begin{array}{cc}
11000 & 01010 \\
10100 & 01001 \\
10010 & 00110 \\
10001 & 00101 \\
10000 & 00010 \\
10000 & 00001 \\
10001 & 00001 \\
10010 & 00010 \\
10100 & 00101 \\
11000 & 01010 \\
\end{array}

\begin{array}{cc}
20000 & 02000 \\
00200 & 00020 \\
00020 & 00002 \\
5 & \\
\end{array}
QM: 2 types of particles

Bosons (integer spin) — 2 or more than one particle can occupy a state

Fermions (half-integer spin) — no more than one particle can occupy a state
In distinguishable particles

Boys:

Fermions:

Note: $\frac{1}{n!} 2^n$ gives $\frac{1}{2!} 5^2 = 12.5$
When "AM" effects become important?

The system should be dense enough.
\# \text{ of available states for each particle} \gg N

then the QM effects are unimportant

\# \gg N
Recall the ideal gas

\[ z_1 = \frac{V T_{ni}}{N_0} \]

but typically it is not very large

\[ \frac{V}{N} \gg \frac{1}{N_0} \]
\( V \gg W \) 

\( \left( \frac{h}{\sqrt{2\pi m h}} \right)^3 \)

An unimportant

Quantum If:

- \( T \) is small
- \( n \) is small
- Density is large
Meaning of the criteria: qm starts to matter when

\[ 3 \sqrt{\frac{N}{\sqrt{N}}} \sim \frac{\text{interparticle distance}}{\text{distance}} \sim \frac{\hbar}{\sqrt{2mE}} \]
Consider a particle with a "typical" energy $E = \hbar \omega$.

its de Broglie wavelength $\lambda$ is

$$\lambda = \frac{\hbar}{p}$$

$$p = \sqrt{2mE} = \sqrt{2m\hbar \omega}$$

$$\lambda = \frac{\hbar}{\sqrt{2m\hbar \omega}}$$
Thus, our criterion means that QM starts to matter when interparticle distance ~ de Broglie's wavelengths start to overlap.