

Physics 341: Problem Set #8
due October 29

You are encouraged to work in groups on these problems, but each student must write up the solutions individually. You must also list your collaborators on your solutions, and cite any external sources you used (other than the course notes or textbook).

I will give partial credit for partial answers, but only if you show your work and explain your reasoning. Be careful with units.

1. Recall that the surface brightness of an exponential disk has the form

$$I(R) = I_0 e^{-R/h_R}$$

where I_0 is the central surface brightness and h_R is the disk scale length. The total brightness is given by integrating this profile from $R = 0$ to $R = \infty$:

$$I_{\text{total}} = \int_0^{\infty} I(R) 2\pi R dR$$

- (a) Show that the total brightness of the exponential disk is

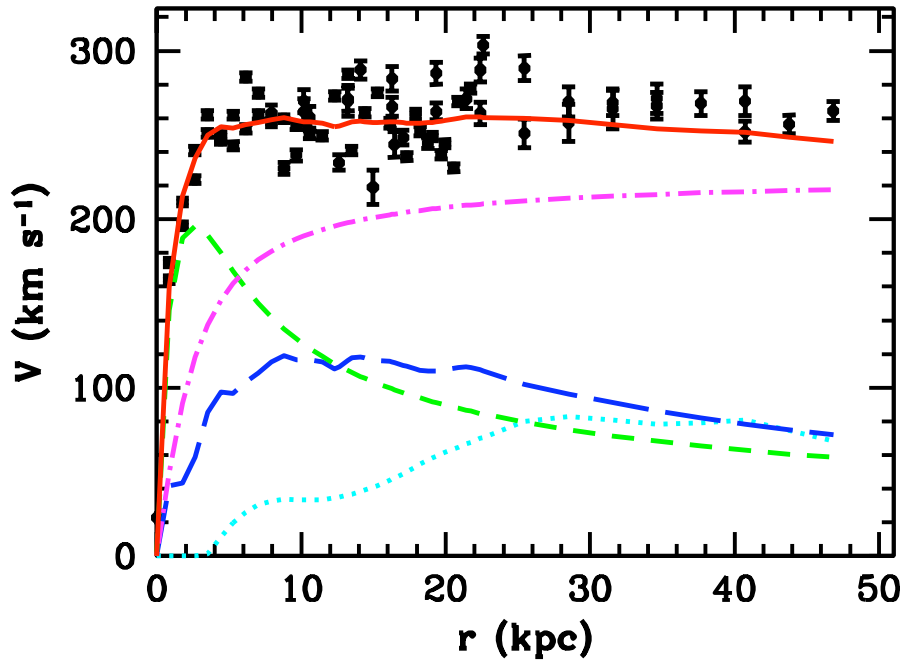
$$I_{\text{total}} = 2\pi I_0 h_R^2$$

Show your work! *Hint:* Let $x = R/h_R$ and rewrite the integral in terms of x and dx . You will need to know that $\int_0^{\infty} x e^{-x} dx = 1$.

- (b) What fraction of the total light is within one disk scale length ($R \leq h_R$)? What fraction of the total light is within three disk scale lengths ($R \leq 3h_R$)? Note that from integration by parts, $\int x e^{-x} dx = -(x + 1)e^{-x}$ plus a constant.

2. On page 2 is reprinted the rotation curve data and model for UGC 5166 that we saw in class. The different components are as follows: short-dash (green) = bulge, long-dash (dark blue) = stellar disk, dotted (cyan) = gas, dot-dash (magenta) = dark matter, solid (red) = total.

- (a) What is the mass of dark matter within 20 kpc?
- (b) What fraction of the total mass within 40 kpc is dark matter?
- (c) What is the total mass of the bulge? *Hint:* check several values of the radius to make sure you've gotten all the enclosed bulge mass.



3. A popular model for the mass distribution in a spheroid (such as a bulge) is the Hernquist model, with density

$$\rho(r) = \frac{M_{\text{total}} a}{2\pi r(a+r)^3}$$

where M_{total} is the total mass and a is a constant with dimensions of length.

- (a) Derive the enclosed mass $M(r)$ and the rotation curve $v(r)$. Show your work.
 - (b) Use the Hernquist model as a model for the bulge component in UGC 5166 above, and plot your derived rotation curve (using any appropriate software). Plot the r axis in kpc, and the v axis in km s^{-1} . To match the bulge rotation curve, Use the total bulge mass you determined above, and use trial and error to find a reasonable value for a .
4. (a) Modified Newtonian Dynamics (MOND) proposes that, for accelerations smaller than some value a_0 , Newton's second law $F = ma$ is replaced by $F = ma^2/a_0$. In that case, what is the rotation curve around a point mass? Work with symbols; there are no numbers to plug in here. *Hint:* the acceleration of an object moving in a circular orbit with radius r and speed v is still $a = v^2/r$.
- (b) Alternatively, some people have proposed to keep $F = ma$ but replace the usual law of gravity $F = GMm/r^2$ with some modified law when the distance becomes larger than some value r_0 . Propose a new law of gravity that can produce flat rotation curves without dark matter. Make sure your modified law has the right dimensions. (There is more than one reasonable answer.)