

**Physics 341: Problem Set #6**  
**due October 15**

You are encouraged to work in groups on these problems, but each student must write up the solutions individually. You must also list your collaborators on your solutions, and cite any external sources you used (other than the course notes or textbook).

I will give partial credit for partial answers, but only if you show your work and explain your reasoning. Be careful with units.

*Important! Don't forget the in-class quiz on Thursday, October 15!* The quiz will be multiple choice, covering concepts, not calculations, and will cover class topics through Lecture 10 (transiting planets). You won't need anything except your brain (and a pencil).

1. In 2003 I helped discover a transiting planet orbiting OGLE-TR-56 (a faint star in the constellation Sagittarius). We used the Keck telescope on Mauna Kea, Hawaii to measure the star's Doppler shift. The radial velocity curve is sinusoidal, with a radial velocity amplitude of  $167 \text{ m s}^{-1}$  and a period of just 1.21 days. During the transits, the planet blocks 1.4% of the star's light. Our best estimate of the star's mass and radius are  $M_{\star} = 1.04 M_{\odot}$  and  $R_{\star} = 1.10 R_{\odot}$ .
  - (a) What is the orbital separation between the star and the planet (in AU)?
  - (b) What are the mass and radius of the planet (in Jupiter units,  $M_J$  and  $R_J$ )?
  - (c) What is the average density of the planet? Is it rocky or gaseous?
  
2. The Kepler space mission was launched by NASA in March, 2009 to look for transiting Earth-mass planets.
  - (a) The geometric probability of having a system oriented just right so that we see a transiting planet is  $p \approx R_{\star}/a$ , where  $R_{\star}$  is the radius of the star and  $a$  is the orbital separation. Kepler is going to observe 100,000 stars; if all of them were just like the Sun, with an Earth orbiting at 1 AU, how many would show transits?<sup>1</sup>
  - (b) Imagine that Kepler discovers the planet "New Earth" orbiting the star "New Sol" — transits in a nearly exact analogue of the Sun/Earth system ( $M_{\star} = 1 M_{\odot}$ ,  $R_{\star} = 1 R_{\odot}$ ,  $M_{\text{planet}} = 1 M_{\oplus}$ ,  $R_{\text{planet}} = 1 R_{\oplus}$ ,  $e = 0$ ,  $a = 1 \text{ AU}$ ). What fraction of New Sol's light is blocked during a transit by New Earth?
  - (c) How often do transits of New Sol by New Earth occur?

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<sup>1</sup>Assume that the orbits are all randomly oriented, so you can just use the probability as given.

- (d) How long does each transit last? Assume that the transit is central, i.e. the projected path of the planet goes right over the center of the star ( $i = 90^\circ$  exactly).
- (e) The reactionary group Just One Earth isn't happy about the discovery of New Earth. They argue that "New Earth" is not a planet, but a white dwarf star instead. White dwarfs can also have a radius like the Earth,  $R_{\text{WD}} = R_{\oplus}$ , and so their transit light curve would look very similar to a planet, but the typical mass of a white dwarf is much higher,  $M_{\text{WD}} = 0.6 M_{\odot}$ . Calculate New Sol's radial velocity amplitude in the two cases when the companion is (1) a planet or (2) a white dwarf (keep the orbital period the same).
- (f) Assuming we can make radial velocity measurements of New Sol with the best precision we have today (approaching the level of  $40 \text{ cm s}^{-1}$ ), will we be able to tell if New Earth is a white dwarf or not?
3. You may have heard that a person falling feet-first into a black hole would be stretched out by the tidal force, in a process affectionately called "spaghettification." But would the effect actually be dramatic? Let's consider:
- (a) What is the tidal force on a person of height  $h$  at the event horizon of a black hole with mass  $M$ ? (You may use Newtonian gravity.)
- (b) It seems reasonable to say that a person would "feel" the stretching only if the tidal acceleration exceeds  $1 \text{ g}$  ( $= 980 \text{ cm s}^{-2}$ ). Find the black hole mass that would produce such a tidal acceleration at the event horizon (for  $h$ , you should use your own<sup>2</sup> height).
- (c) Use your results from (a) and (b) to say whether you would be spaghettified by the black hole at the center of the Milky Way ( $M = 4 \times 10^6 M_{\odot}$ ).
- (d) What about by the black hole in the binary system M33-X7 ( $M \approx 16 M_{\odot}$ )?

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<sup>2</sup>For those about to be spaghettified, we salute you. Sorry about this unfortunate fate, but it's all in the name of science!