

**Physics 341: Problem Set #2**  
**due September 17**

You are encouraged to work in groups on these problems, but each student must write up the solutions individually. You must also list your collaborators on your solutions, and cite any external sources you used (other than the course notes or textbook).

I will give partial credit for partial answers, but only if you show your work and explain your reasoning. Be careful with units. Don't neglect problem 3 (on the back)!

1. We showed that for circular orbits, Kepler's Third Law can be written as

$$P^2 = \frac{4\pi^2}{GM} r^3$$

where  $M$  is the mass of the central object.

- (a) Use this expression and a Taylor expansion to derive the following approximation for the orbital period of a satellite in "low Earth orbit", with a constant height  $h$  above the surface of the Earth, assuming that  $h \ll R_{\oplus}$ :

$$P \approx P_0 \left( 1 + \frac{3h}{2R_{\oplus}} \right)$$

- (b) What is the constant  $P_0$  (in symbols), and what is its value (in minutes)?
- (c) The Hubble Space Telescope orbits the Earth at an altitude of  $h = 560$  km. Compare its exact orbital period based on Kepler's Law with your approximation.
2. You are on a space mission to land on various asteroids in the solar system. After many long years in cramped quarters and some "misunderstandings" with your crewmates, you've developed a nagging suspicion that they are planning to leave you behind on the next asteroid.<sup>1</sup> You've decided that you're only going out there again if you can jump off the asteroid under your own power. Can you do it? Let's find out...
- (a) Estimate the velocity you achieve when you jump straight up. *Hint:* Use the height you reach jumping on Earth to estimate the change in your potential energy, and then use conservation of energy to estimate your initial kinetic energy.
- (b) Now estimate the radius of the largest asteroid you could escape from by jumping. (You will need to make an assumption about the mean density of asteroids, which are made of rock and ice; just explain your reasoning.)

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<sup>1</sup>Just because you are paranoid doesn't mean they *aren't* after you.

3. We will see ellipses a few times this semester; remember back to high school that the equation for an ellipse centered at the origin (see figure below) is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Show that the area of the ellipse is  $A = \pi ab$ . *Hint:* Work out the area of just one quadrant of the ellipse (the shaded area, for example) and use symmetry to get the total area. It may help you to know the following integral:  $\int_0^1 \sqrt{1-u^2} du = \pi/4$ .

