

Physics 341: Problem Set #1
due September 10

You are encouraged to work in groups on these problems, but each student must write up the solutions individually. You must also list your collaborators on your solutions, and cite any external sources you used (other than the course notes or textbook).

I will give partial credit for partial answers, but only if you show your work and explain your reasoning. Be careful with units.

1. (a) Use dimensional analysis to derive a relationship between the total mass of a gravitationally bound system M , its typical size R , and the typical speed v of its components.
(b) Using your analysis above, estimate how many stars are in a typical globular cluster with $R = 10$ pc in which the average stars are moving around at $v = 20$ km s⁻¹. You may need to make an assumption about the average star; be sure to state any assumptions explicitly.
(c) A cluster of galaxies has a mass of $10^{14} M_{\odot}$ and a size of 1 Mpc (a megaparsec = 10^6 pc). How fast do galaxies move around in the cluster? As above, astronomical speeds are usually given in units of km s⁻¹.
2. The Universe is about 14 billion years old and has been expanding under the influence of gravity. Use dimensional analysis to estimate the average density of the Universe. About how many hydrogen atoms are there in 1 cubic meter of “empty” space?
3. I study exploding stars called supernovae. It turns out that explosions in space and explosions on the ground can be understood in a very similar way.
 - (a) Use dimensional analysis to estimate the size R of a blast wave at a time t after an explosion with energy E , propagating into a medium of ambient density ρ . *Hint:* Those are all the quantities you need; gravity is not directly relevant here.
 - (b) Information about the first atomic bomb tests was kept very secret. However, physicist George Taylor derived the same result you just did, and figured out the (highly-classified) energy of one of the tests based on just a published photograph showing a fireball expanding through the air, 100 m in size just 0.02 s after the explosion. What was the energy (in ergs)? What mass (in grams) was converted into energy ($E = mc^2$)? *Hint:* You will need to look up the density of air.
 - (c) How large (in light-years) would you estimate the remnant of a supernova ($E = 10^{51}$ erg) would be 1000 years after the explosion, as it expands into the interstellar medium with a typical density of 1 hydrogen atom per cubic centimeter?